

Spatial estimation of wind states from the aeroelastic response of a wind turbine

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Abstract

We formulate an observer of state variables describing the spatial distribution of the wind incident over the rotor of a wind turbine.

The present approach exploits the idea that the wind turbine response, if measured in terms of a sufficiently rich set of states, contains not only information regarding the temporal variation of the wind, but also of its spatial distribution over the rotor disk. In this sense, the whole rotor is used as a sophisticated sensor that is capable of providing information on the wind field which is far more complete than the information available by the use of standard on-board anemometers.

The formulation first estimates structural states describing the flexible response of tower and blades, and then feeds this information to the wind estimator. In turn, the instantaneous estimates of the flexible states of tower and blades and of the spatial wind distribution can be used for enabling sophisticated individual-blade fatigue and load alleviating control laws, which are however not discussed here.

The proposed procedures are tested with the help of numerical experiments conducted using a high-fidelity aero-servo-elastic simulator, and some preliminary conclusions regarding the efficacy and limitations of the approach are drawn.

Keywords: wind turbine; control; aero-servo-elasticity; Kalman filter.

1 Introduction

In this paper we formulate an observer of the wind blowing on the rotor of a wind turbine, to be used for enabling the implementation of advanced control laws. The work is motivated by the idea that knowledge of the instantaneous wind-over-the-rotor distribution, if it were available, could be exploited so as to improve the performance of pitch and torque

control laws. For example, if one knew the current wind vertical and horizontal shear values, one could try to exploit this information for reducing the oscillatory loads on the machine [6].

Unfortunately, information regarding the spatial distribution of the wind can not be obtained by using standard on-board anemometers, which can only provide mean hub-height wind values, nor it can be easily obtained by other practical means. Although wind observers have been previously described in the literature [9], these provide only hub-height estimates, i.e. constant-over-the-rotor wind values. While this information is useful and can be profitably used, for example for scheduling the control gains in terms of the wind speed, it does not account for important effects on the aeroelastic response of the wind turbine due to wind non-uniformity. The proposed wind observer tries to remove these limitations, by reconstructing a more complete picture of the instantaneous wind-over-the-rotor distribution.

In the proposed approach, estimates of states expressing the wind spatial distribution are obtained by using a cascading series of Kalman filters [5]. A first filter is responsible for the reconstruction of the tower states, while a second set of parallel filters (one for each blade) operates in series to the tower filter and reconstructs the flexible blade states. These filters are used for observing structural states of the wind turbine, describing its aeroelastic response; in turn, the structural response information is fed to a final filter whose role is to interpret the wind turbine response and estimate the wind in terms of suitable states.

In this work, the spatial wind distribution is approximated using a simple model that accounts for mean hub wind, vertical and lateral shear, wind direction and vertical wind component. These unknown wind parameters are promoted to the role of dynamical states and are identified on-line by filtering. To this end, we use a wind turbine reduced

model that accounts for the coupled dynamic equilibrium of drive-train, tower fore-aft and side-side motion, and elastic blade motion. At each instant of time, an adaptive extended Kalman filter estimates the wind states by enforcing in a stochastically optimal sense the satisfaction of the reduced model dynamic equilibrium equations. This is obtained by regarding the wind states as the sole unknowns of the model governing equations, whereas all control inputs and states are either available by readings of the on-board sensors or known through the estimates provided by the structural flexible state observers.

The proposed methodology is demonstrated by using a high fidelity simulation environment, that includes an aero-servo-elastic wind turbine multi-body model of the plant, models of the wind, of the sensors, of the actuators and of measurement and process noise. Extensive simulations in gusty and turbulent wind conditions have been conducted to verify the ability of the proposed observers to identify the flexible response of the machine and the spatial characteristics of the wind. The experiments conducted so far highlight the potential of the proposed approach, and at the same time clearly indicate critical areas where substantial improvements should be made before the technique can be validated and deployed in the field.

2 Observer of Wind States

In this work, we consider the architecture of state estimators shown in Fig. 1.

A first observer reconstructs the flexible states $\mathbf{x}_t = (\mathbf{q}_t^T, \dot{\mathbf{q}}_t^T)^T$ of the wind turbine tower, represented by the amplitudes \mathbf{q}_t of a suitable modal basis and their rates $\dot{\mathbf{q}}_t$. These quantities are estimated using a Kalman filter [12, 11] on the basis of readings provided by accelerometers and strain-gages placed along the tower height. In the figure and later on in this document, estimated or measured quantities are indicated with the notation $\hat{(\cdot)}$. Similarly to the tower case, a set of observers, operating in parallel on each blade and in series with respect to the tower observer, estimate the blade states $\mathbf{x}_{b_i} = (\mathbf{q}_{b_i}^T, \dot{\mathbf{q}}_{b_i}^T)^T$ in terms of modal amplitudes \mathbf{q}_{b_i} and their rates $\dot{\mathbf{q}}_{b_i}$ using accelerometer and strain-gage measurements. To keep the number of necessary sensors to a minimum, which is important for reasons of simplicity, the filters can operate with one accelerometer and one strain gage for the tower and for each blade, which is a practical configuration already available on several wind turbines. To reduce the need for tuning of the covariance matrices of the various fil-

ters, an adaptive filtering [10] approach is used, that reconstructs the noise statistics by keeping in memory a buffer of past values. The observers of the structural states are described in greater detail in Ref. [5].

As shown in the figure, the estimated states are fed directly to the controller, which can explicitly account for this information in its formulation, but they are also inputted in a wind estimator. This third estimation stage is responsible for providing estimates of suitably chosen wind states, and uses a wind turbine reduced model as described in the following.

A dynamic model of a wind turbine can be expressed in terms of its governing equations as

$$\dot{\mathbf{x}}_{\text{wt}} = \mathbf{f}_{\text{wt}}(\mathbf{x}_{\text{wt}}, \mathbf{u}_{\text{wt}}, \mathbf{w}), \quad (1)$$

where \mathbf{x}_{wt} are wind turbine states, \mathbf{u}_{wt} is the wind turbine input vector, and $\mathbf{w} = \mathbf{w}(\mathbf{r}, t)$ the wind vector field which depends on the space location \mathbf{r} and on time t . The state vector \mathbf{x}_{wt} is

$$\mathbf{x}_{\text{wt}} = (\theta, \dot{\theta}, \psi, \dot{\psi}, \dots, \beta_i, \dot{\beta}_i, \dots, \mathbf{q}_t^T, \dot{\mathbf{q}}_t^T, \dots, \mathbf{q}_{b_i}^T, \dot{\mathbf{q}}_{b_i}^T, \dots, T_g)^T, \quad (2)$$

$i = (1, B)$, where θ is the yaw angle, ψ the rotor azimuth, β_i the pitch of the i th blade, T_g the generator torque and B the number of blades, whereas the input vector is

$$\mathbf{u}_{\text{wt}} = (\theta_c, \dots, \beta_{i_c}, \dots, T_{g_c})^T, \quad (3)$$

$i = (1, B)$, where θ_c is the commanded yaw angle, β_{i_c} the commanded blade pitch setting and T_{g_c} the commanded generator torque. At each instant of time the states of vector (2) are known, either from sensors or from the observers described above. Similarly, at each instant of time also the inputs appearing in vector (3) are known as computed by the on-board controller.

The key idea pursued in this work is that, by taking advantage of the fact that both states and inputs are known in (1), one can infer the wind \mathbf{w} blowing on the rotor. To this end, it is first necessary to introduce a spatial discretization of the wind. A possible solution is provided by the frequently used wind model given by

$$\mathbf{w}(\mathbf{r}, t) = \left(V_h(t) \left(1 + \frac{z-H}{H} \right)^{V_s(t)} + V_{ls}(t) \frac{z-H}{2R} + H_{ls}(t) \frac{y \cos \alpha(t) - x \sin \alpha(t)}{2R} \right) \mathbf{i}_v(t) + V_z(t) \mathbf{i}_3, \quad (4)$$

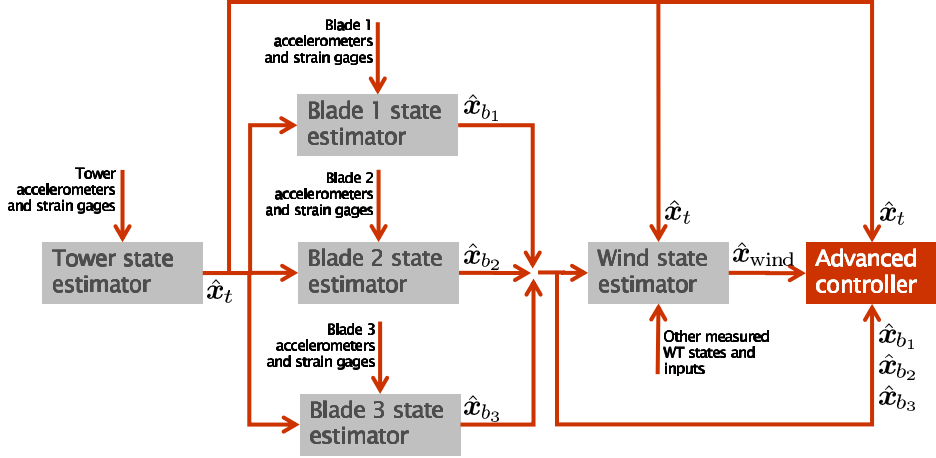


Fig. 1: Overall architecture of the estimators of flexible tower and blade states, and of wind states.

where \mathbf{i}_v is the wind direction unit vector, \mathbf{i}_3 is the vertical unit vector, $\mathbf{r} = (x, y, z)^T$ are coordinates in the tower root reference frame, V_h is the horizontal wind speed at the height $z = H$ of the rotor hub, V_s is the vertical-shear power law exponent, V_{1s} and H_{1s} are the vertical and horizontal, respectively, linear shear coefficients, R being the rotor radius, and α the horizontal wind direction.

Selecting as unknown wind parameters the hub wind V_h , either the power law exponent V_s or the vertical shear coefficient V_{1s} , the horizontal wind shear coefficient H_{1s} , the relative wind direction α and the vertical wind component V_z , we can define the vector

$$\mathbf{x}_{\text{wind}}(t) = (V_h(t), V_s(t) | V_{1s}(t), H_{1s}(t), \alpha(t), V_z(t))^T, \quad (5)$$

and write the wind vector field \mathbf{w} as a function of \mathbf{x}_{wind} , i.e. $\mathbf{w}(\mathbf{r}, t) = \mathbf{w}(\mathbf{x}_{\text{wind}}(t))$.

Next, the unknown wind parameters \mathbf{x}_{wind} are promoted to the role of states; this transforms their estimation problem into a new state estimation one. The governing wind estimation equations in state-space form are written as

$$\dot{\mathbf{x}}_{\text{wind}} = \mathbf{m}_{\text{wind}}, \quad (6a)$$

$$\mathbf{y}_{\text{wind}} = \mathbf{h}_{\text{wind}}(\mathbf{x}_{\text{wind}}, \mathbf{u}_{\text{wind}}), \quad (6b)$$

$$\mathbf{z}_{\text{wind}} = \mathbf{y}_{\text{wind}} + \mathbf{n}_{\text{wind}}. \quad (6c)$$

Equations (6a) represent the wind parameter dynamics evolution equations, where a process noise term \mathbf{m}_{wind} to the right hand side is responsible for exciting the temporal variations of the wind

states \mathbf{x}_{wind} . Equations (6b) are the output definition equations, representing the residuals of equations (1):

$$\mathbf{h}_{\text{wind}}(\mathbf{x}_{\text{wind}}, \mathbf{u}_{\text{wind}}) = (\dot{\hat{\mathbf{x}}}_{\text{wt}} - \mathbf{f}_{\text{wt}}(\hat{\mathbf{x}}_{\text{wt}}, \mathbf{u}_{\text{wt}}, \mathbf{w}(\mathbf{x}_{\text{wind}})))_{\text{dyn eq}}. \quad (7)$$

Notice that the residuals are evaluated for the known wind turbine state $\hat{\mathbf{x}}_{\text{wt}}$ and known input \mathbf{u}_{wt} vectors, which can therefore be considered as inputs to the wind state-space model, i.e. $\mathbf{u}_{\text{wind}} = (\hat{\mathbf{x}}_{\text{wt}}^T, \mathbf{u}_{\text{wt}}^T)^T$. Furthermore, of the complete set of governing equations (1) represented by dynamic equilibrium and kinematic equations, only the former set depends on the wind field \mathbf{w} ; therefore, in (7), only the dynamic equilibrium set of equations enters into the definition of the outputs. Finally, equations (6c) represent the measurement definition equations. For this special problem, the measurements at all time instants are simply

$$\mathbf{z}_{\text{wind}} = \mathbf{0}, \quad (8)$$

which means that we are trying to enforce a null error in the satisfaction of the dynamic equilibrium equations of the wind turbine model. In other words, we are trying to find those wind states that satisfy the dynamic equilibrium equations when these are evaluated in terms of the given measured and reconstructed quantities. Notice that a noise term \mathbf{n}_{wind} appears in the measurement definitions; however, in this case this is not a proper measurement noise term but in reality a process noise one, since it appears as an additive term in the dynamic equilibrium equations.

The state estimation problem (6) is solved here again with an adaptive extended Kalman filter.

3 Results

We begin by illustrating the performance of the observers of structural flexible states, by considering the response of the LTW62 wind turbine [2]. The tests are conducted within a simulation environment, more fully described in Ref. [4]. The virtual plant includes a detailed aero-servo-elastic model of the wind turbine, models of the wind, of the sensors and of the actuators on-board the machine, and it is operated by a control system that includes supervision of operating conditions and active blade pitch-torque control based on a collective LQR formulation [4]. The aero-servo-elastic model of the wind turbine is implemented with the software `Cp-Lambda` (`Code for Performance, Loads and Aeroelasticity by Multi-Body Dynamic Analysis`), based on a finite-element multibody formulation described in Ref. [3] and references therein.

The tower has a strain gage located at $\xi_Q = 25$ m ($L = 58.27$ m), and an accelerometer at its top in the nacelle. Each blade has a strain gage at $\xi_Q = 25$ m ($R = 30.08$ m), and an accelerometer at $\xi_P = 5$ m. All sensors are affected by errors modeled as white noise with an amplitude not to exceed the 5% of the measured signal. Sensor signals are filtered with a 2 Hz pass-band low-pass filter. The adaptive Kalman filters use buffers of 15 past samples to reconstruct on-line the noise statistics.

At first, we consider the case of the extreme operating gust (EOG) at 15 m/sec [1]. Figure 2 shows the time history of the wind, and the resulting rotor angular speed and collective blade pitch (respectively, from top to bottom). Figure 3 shows, at top, the time history of the blade deflection at the accelerometer location, and at bottom the time history of the tower tip deflection. The “true” plant response is reported using a solid line, while the observed one is shown using a dashed line; both quantities were non-dimensionalized by the maximum value achieved by the plant response. The peaks in the blade response show some over-prediction, but overall, the quality of the reconstructed response seems to be satisfactory and adequate for control purposes.

Next, we evaluate the performance of the observer in turbulent wind conditions. Transient simulations were conducted for a duration of 600 sec with constant mean hub-height wind speed and Category A turbulence. Figure 4 shows the time history at the hub of the turbulent wind in the case of 16 m/sec

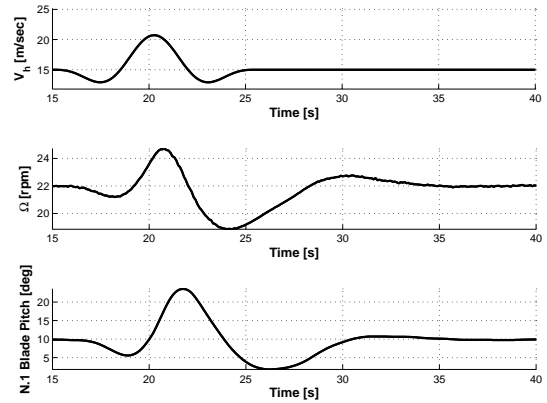


Fig. 2: EOG at 15 m/sec. From top to bottom, time history of wind, rotor angular speed and collective blade pitch.

mean wind, and the resulting rotor angular speed and collective blade pitch (respectively, from top to bottom). Figure 5 reports at top the blade non-dimensional deflection time history, and at bottom the tower tip non-dimensional deflection time histories. As in the previous plots, the plant response is reported using a solid line, while the observed response is shown using a dashed line. The observed deflections match quite well the true ones throughout the entire simulation, with little delay and good overall shape of the curves. As in the EOG case, the tower response exhibits a higher accuracy than the blade response, with peaks that are nicely captured.

Having assessed the quality of the structural state estimates, we turn to the estimation of the wind states.

At first, we consider the case where both plant and reduced model are implemented with the `SymDyn` software [13]. Time histories of wind states were generated and used for calculating the response of a `SymDyn` model of the LTW62 wind turbine, here again operating in closed loop with the collective LQR controller of Ref. [4]. Horizontal, vertical and lateral wind components were generated using the Kaimal turbulence model, while the vertical shear power law exponent and the vertical and horizontal wind shear coefficients were varied according to assumed deterministic time histories.

Notice that, in this example, there is no mismatch between plant model and observer model. Consequently, discrepancies between the actual and reconstructed wind states are only due to the presence of noise in the procedure. These preliminary simulations were conducted so as to verify the correct implementation of the software and determine the actual observability of the wind states.

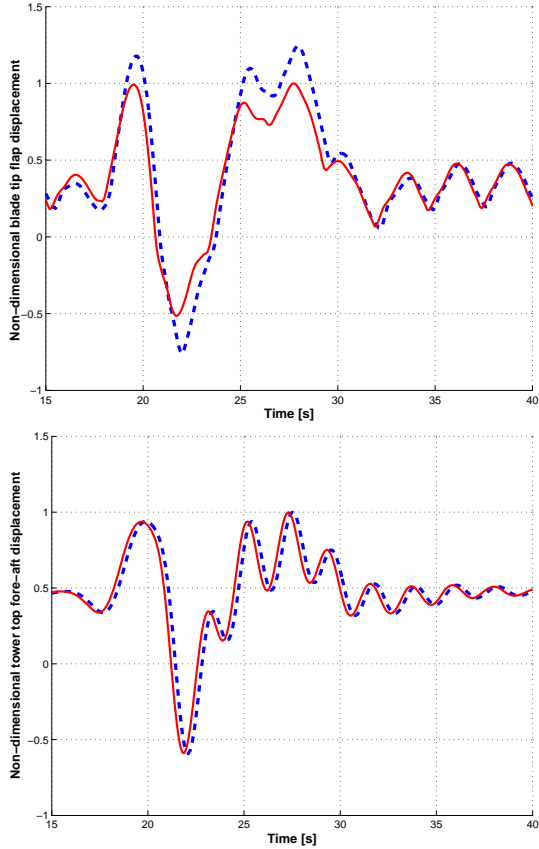


Fig. 3: EOG at 15 m/sec. Time history of non-dimensional blade (top) and tower (bottom) deflections. Plant response: solid line; observed response: dashed line.

Figure 6 shows the time history of the horizontal hub wind state V_h ; the actual wind is reported with a solid line, while the reconstructed one using a dashed line. Figure 7 shows at top the reconstruction of the linear vertical shear coefficient V_{ls} , and at bottom the reconstruction of the linear horizontal shear coefficient H_{ls} . The observer seems capable of capturing quite well the hub wind with its turbulent fluctuations. The filter warm up time is extremely rapid, and barely visible in the figures. The two shears are also reconstructed with reasonable accuracy, although it is interesting to observe that the turbulent fluctuations of the wind seem to pollute much more the observed horizontal wind state than the vertical one. On the other hand, the latter, although less affected by the wind fluctuations, is characterized by a somewhat larger delay.

Several tests were conducted in different wind conditions, with a different number and choice of unknown wind states, and for varying levels of noise. The results of these numerical experiments showed

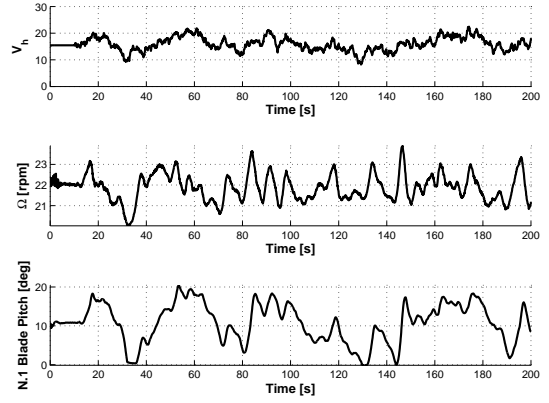


Fig. 4: Turbulent wind with 16 m/sec mean hub-height wind speed and Category A turbulence. From top to bottom, time history of wind at hub, rotor angular speed and collective blade pitch.

that, although all wind states are observable at low noise levels, some of the states are more reliably reconstructable than others. This behavior manifests itself with a progressively less faithful reconstruction with increasing noise levels in the system, with the observer that finds progressively more difficult to distinguish the contribution of each different wind state within the wind turbine response. From the tests conducted so far, it appears that the wind states that represent rotor in-plane wind components are the most difficult to reconstruct in a reliable manner. These qualitative results were corroborated in a more quantitative manner by verifying the observability of the wind states using the Fisher Information Matrix (FIM) approach [7]. The test measures the covariance of the error in the estimate of each given set of wind states, large values of the covariance indicating a low information content in the data.

Next, we consider the more realistic case when there is a mismatch between plant and reduced model. The plant is realized as an aero-servo-elastic model implemented in `Cp-Lambda`, with tower and blades modeled using geometrically exact non-linear beams discretized in space using the finite element method. The tower and blade responses are estimated using the structural observers. The wind observer is based on the same `SymDyn` reduced model used for the previous tests.

Before attempting the reconstruction of wind states, we verified the similarity of the response of the `Cp-Lambda` and `SymDyn` models, when excited by the same wind conditions. Clearly, this is a crucial check: since the reduced model is used for in-

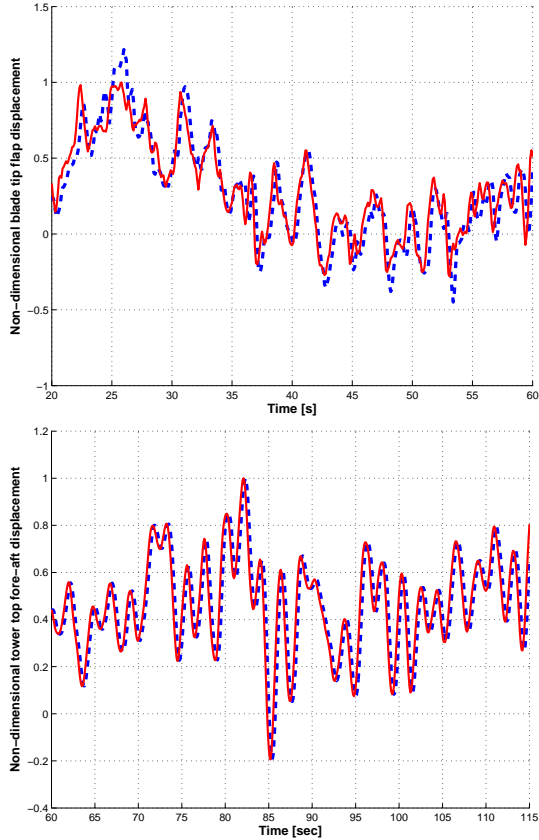


Fig. 5: Turbulent wind with 16 m/sec mean hub-height wind speed and Category A turbulence. Time history of non-dimensional blade (top) and tower (bottom) deflections. Real response: solid line; observed response: dashed line.

interpreting the response of the plant and extracting from it estimates of the wind states, if the two models respond differently to the same excitation, the wind reconstruction will be negatively affected. The tests that we conducted, and that are not reported here for brevity, showed that the two models have quite dissimilar responses when subjected to rotor in-plane wind components. This behavior was interpreted as due to the rather crude modeling assumptions in *SymDyn*, which models the flexibility of the blade with a single flapping degree of freedom using an equivalent hinge. Given the different behavior of the *Cp-Lambda* and *SymDyn* models in these conditions, and at the light of the observation made above about the difficulty of reconstructing rotor in-plane wind states even in the absence of model mismatch, we concentrate in the following on the sole observation of the hub wind and horizontal and vertical shears.

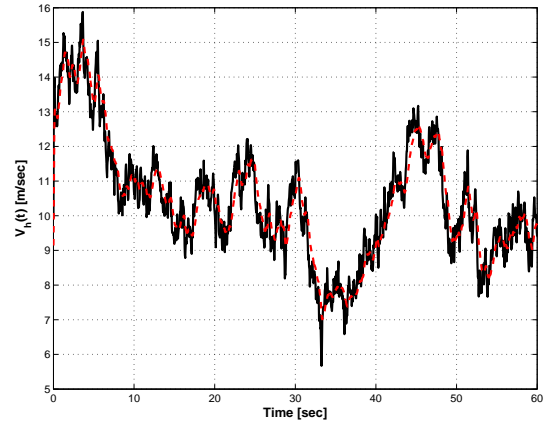


Fig. 6: Estimation of the hub wind state V_h , for the case with no model mismatch. Real: solid line; observed: dashed line.

Of the extensive testing that was conducted in a variety of conditions, we show here just a few results, which however illustrate well the behavior of the proposed observation system.

At first, we consider a gust that affects both the horizontal speed V_h and the linear horizontal shear coefficient H_{ls} . The results are shown in Fig. 8. There is a reasonable accordance between the real and observed wind state values, even if the observed signals are here noisier than in the previous examples. The overall shape of the response is captured with sufficient accuracy, although the observed response is delayed of about 1 sec, and peaks are underestimated. The warm up time seems to be significantly higher than before, and the filter appears to reach steady state after about 5 sec. Before and after the gust, the estimated response shows small off-sets; furthermore, it also appears that the periodicity of the aeroelastic response of the plant, due to the non-uniform wind blowing over the wind turbine rotor, induces a periodic error in the wind estimates, which is particularly apparent in the hub wind reconstruction. This behavior is due to subtle differences in the response of the two models; here again, improvements are possible only by using a more faithful reduced model, i.e. by reducing the model mismatch.

Next, we consider the turbulent wind case. Figure 9 shows the observer performance when estimating the horizontal component V_h of a turbulent wind of Category A intensity with a mean value of 21 m/sec. Here again the general trend is captured, although the higher frequency fluctuations are missed. Similar results have been obtained for several mean hub wind speeds.

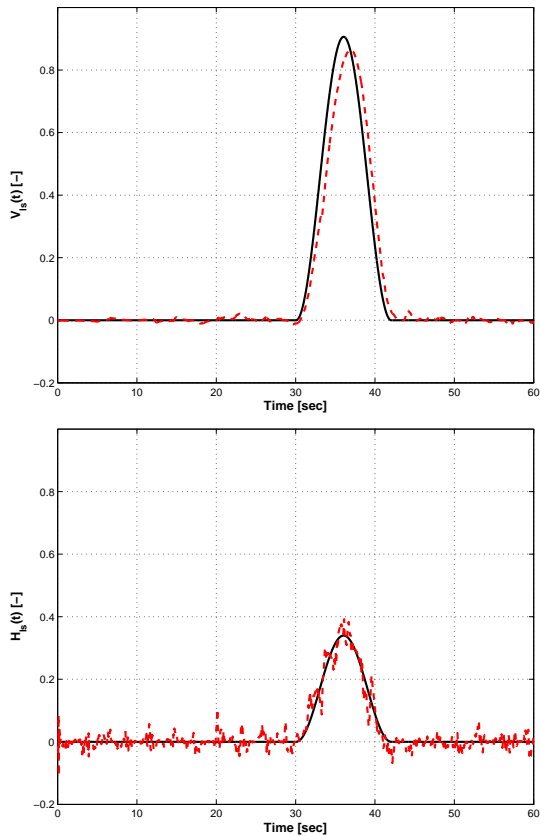


Fig. 7: Estimation of the vertical linear shear state V_{ls} (top) and horizontal linear shear state H_{ls} (bottom), for the case with no model mismatch. Real: solid line; observed: dashed line.

4 Conclusions

In this work we have formulated a wind observer that, by using the response of a wind turbine, estimates the spatial distribution of the wind over the rotor disk using a Kalman filter. This information can be used for designing advanced control laws [6] that respond to variations in the spatial distribution of the wind, for example self-reacting to changes in the vertical wind profile or other effects. It is clear that such information, if of sufficient quality and if properly used, can in principle enable the implementation of load reduction and fatigue damage alleviation strategies.

Although the basic idea behind wind state estimation has been successfully demonstrated by the tests conducted in the absence of model mismatch, there is important work that still needs to be performed before the system is mature enough to be deployed in the field. A fundamental remark that should be

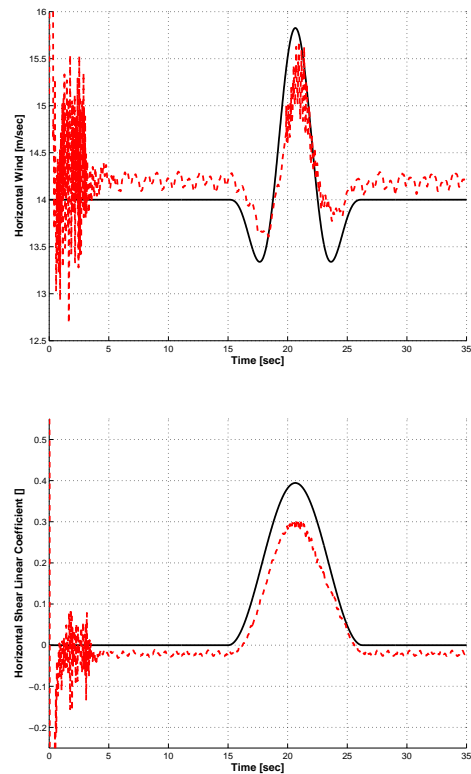


Fig. 8: Estimation of generalized wind states with cascading structural and wind observers. Deterministic wind. Hub wind V_h (top) and horizontal linear wind state H_{ls} (bottom). Real: solid line; observed: dashed line.

made is that the quality of the wind estimates can only be as good as the faithfulness of the reduced model to the response of the plant. In fact, the reduced model is used for interpreting the response of the plant and extracting from it the hidden information about the spatial distribution of the wind over the rotor; therefore, any mismatch between plant and reduced model invariably translates into errors in the wind estimates.

The quality of the results obtained so far seems to be constrained by the limits of the reduced model that was used in the current implementation, and that employs a rather crude modeling of the blade and tower elastic responses. Differences in the response excited by rotor in-plane wind components prevent the estimation of these wind states all together. For the other wind states the situation is less critical, but nonetheless differences with respect to the plant response induce off-sets and oscillations in the estimates even in steady wind conditions. Clearly, these problems can only be exacerbated in

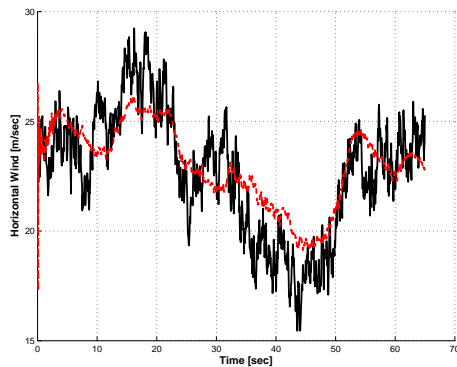


Fig. 9: Estimation of generalized wind states with cascading structural and wind observers. Category A turbulence. Hub wind V_h . Real: solid line; observed: dashed line.

gusty or turbulent winds.

Generally speaking, with the current system there is a growing difficulty in obtaining good quality estimates when trying to observe an increasing number of wind states. This is particularly apparent when turbulence is involved. Here again, this behavior can be traced back to a general lack of fidelity of the reduced model.

The only solution to alleviating the limitations that the current wind state observer has shown seems to be in the adoption of a more faithful reduced model. At present we are implementing a version of the filter based on the software **Fast** [8], that should provide noticeable improvements due to its model-based modeling of blades and tower.

Acknowledgements

The authors acknowledge the contribution of D. Devecchi, V. Ronchi and B. Savini in the implementation of the software described in this document and in the development of some of the examples. Data for the modeling of the LTW62 wind turbine was provided courtesy of the LeitWind company.

References

- [1] Anonymous, Wind Turbines — Part 1: Design Requirements, Ed. 3.0, *International Standard IEC 61400-1*, 2005.
- [2] LeitWind S.p.A., Via Brennero 34, I-39049 Vipiteno, BZ, Italy, www.leitwind.com.
- [3] Bauchau, O.A., Bottasso, C.L. and Nikishkov, Y.G., “Modeling Rotorcraft Dynamics with Finite Element Multibody Procedures”, *Mathematics and Computer Modeling*, **33**:1113–1137, 2001.
- [4] Bottasso, C.L. and Croce, A., “Advanced Control Laws for Variable-Speed Wind Turbines and Supporting Enabling Technologies”, Scientific Report DIA-SR 09-01, Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy, January 2009.
- [5] Bottasso, C.L. and Croce, A., “Cascading Kalman Observers of Structural Flexible and Wind States for Wind Turbine Control”, Scientific Report DIA-SR 09-02, Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy, January 2009.
- [6] Devecchi, D., *Tecniche di Controllo Aeroelastico con Passo Individuale delle Pale per Aerogeneratori* (in Italian), M.Sc. Thesis, Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy, 2009.
- [7] Jategaonkar, R.V., *Flight Vehicle System Identification. A Time Domain Approach*, AIAA Progress in Astronautics Aeronautics, Reston, VA, 2006.
- [8] Jonkman, J., NWTC Design Codes, <http://wind.nrel.gov/designcodes/simulators/fast/>, last modified August 12, 2005; accessed August 12, 2005.
- [9] Ma, X., *Adaptive Extremum Control and Wind Turbine Control*, Ph.D. Thesis, Technical University of Denmark, 1997.
- [10] Myers, K.A. and Tapley, B.D., “Adaptive Sequential Estimation with Unknown Noise Statistics”, *IEEE Transactions on Automatic Control*, **21**:520–523, 1976.
- [11] Reif, K. and Unbehauen, R., “The Extended Kalman Filter as an Exponential Observer for Nonlinear Systems”, *IEEE Transactions on Signal Processing*, **47**:2324–2328, 1999.
- [12] Simon, D., *Optimal State Estimation: Kalman, H-infinity, and Nonlinear Approaches*, John Wiley & Sons, Inc., 2006.
- [13] Stol, K. and Bir, G., NWTC Design Codes, <http://wind.nrel.gov/designcodes/simulators/symdyn/>, last modified May 26, 2005; accessed August 18, 2008.