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HELICOPTER TAILROTOR DYNAMICS CHARACTERIZATION AND STABILITY BY MULTIBODY/MULTIDISCIPLINARY ANALYSIS

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Abstract. *The multibody approach allows to use modular models to build complete detailed systems. The size of the resulting model may become soon very large; its complexity is dictated by the complexity of the problem and by the advantages of a generic approach. The capability to obtain a dynamic charaterization and to analyze the local stability of periodic orbits is of paramount importance, especially in industrial applications. This paper presents all the hurdles which may be encountered while trying to extract eigensolutions for a multibody model based on a redundant coordinates set index three DAE system formulation. This paper applies an effective method to overcome these limitations, based on Proper Orthogonal Decomposition (POD), to the analysis of a helicopter tailrotor. Data analysis using POD is conducted to extract a set of basis functions, called Proper Orthogonal Modes (POM) from numerical simulations, for subsequent use in a* Galerkin *projection that yields low-dimensional dynamical models. The POMs are a minimal set of output signals that can be used to identify the dominant eigenvalues of the transition matrix. Finally the same methodology is applied to the periodic stability investigation recurring to the construction of stroboscopic* Poincaré *maps.*

1 INTRODUCTION

In the last decade, the continuous increase in computer power allowed to simulate complex dynamical systems using large scale nonlinear models. This resulted in the exploitation of redundant coordinate set approaches to the analysis of problems which previously required a large amount of simulations by means of dedicated analysis formulations. A typical application that requires sophisticated analysis capability is the design of complex deformable servoaeromechanical systems, like helicopter rotors and tiltrotors. Their analysis can be effectively conducted by means of general purpose modeling codes based on multidisciplinary methodologies. From the structural mechanics point of view, the need to account for large rotations and displacements naturally leads to the use of multibody analysis codes; these on turn may allow the simultaneous analysis of multidisciplinary problems, provided their formulation meets general purpose requirements. This is particularly true for helicopters and tiltrotors; in fact, great impulse to the development of specific fields of multibody analysis originated from the needs of deformable rotor dynamics and aeroelasticity; at the same time, the multibody analysis approach is gaining acceptance in the rotorcraft industry.

The multibody approach allows the use of modular models to build a complete system with the required detail for each subpart. As a result, the same model (or common components) can be used for the different analyses that are required at each stage of the development. One of the most significant features is that kinematics have an exact formulation within the chosen idealization of the real system. By means of these codes the designer can progress from simple rigid body models up to fully detailed analysis, in which an accurate and nonlinear description of constraints, deformable elements, servohydraulic circuits, aerodynamic forces and control system components can be introduced[1]. The size of the resulting model, mainly pushed by the increasing demand for details in deformable structure modeling, may soon become very large; the problem increases by orders of magnitude when detailed free-wake aerodynamics are included.

The complexity of the model is dictated by the complexity of the problem and by the advantages of a generic approach; however, to make such large models useful in the design and analysis phases, a methodology capable of synthesizing the significant results in each condition is required. This need is particularly strong when synthesizing global dynamical properties and when assessing the stability of a system. The capability to analyze the local stability of periodic orbits is of paramount importance, especially in industrial applications, even if it may not be sufficient to assess the global stability of a system.

It can be investigated using *Floquet* theory, originally developed for the stability analysis of the solution of linear ordinary differential equations with periodic coefficients[2]. The stability of the system can be inferred from the spectral radius of the monodromy matrix, which is the transition matrix that relates two periodic solution state vectors separated by a time period. In classical applications of this method, the entire monodromy matrix is computed first; then its eigenvalues are evaluated. There are different methods to compute the monodromy matrix, but all of them are somewhat impractical to pursue for large numerical models, so successful applications of this approach are limited to systems with relatively few states.

The computation of the monodromy matrix and of its eigenvalues can be extremely demanding, both in terms of computational power and memory requirements. An alternative, convenient way to study the stability of periodic orbits is based on the construction of stroboscopic *Poincaré* maps, which represent the discrete map obtained by sampling the state vector every period. An empirical method to reconstruct a local jacobian of the *Poincaré* map associated with the continuous time system, from data obtained either by experiments or by simulations, has been presented by Murphy *et al*[3]. Despite being very interesting, this method, based on a least-square identification of the jacobian matrix, might quickly become unmanageable, as the number of degrees of freedom used to represent the system increases.

This paper presents an effective method to overcome these limitations, based on *Proper Orthogonal Decomposition* (POD), also known as *Karhunen-Loéve Decomposition*[4]. POD is a powerful and elegant method of data analysis, aimed at obtaining a low-dimensional approximate description of high-dimensional processes. Data analysis using POD is conducted to extract a set of basis functions, called Proper Orthogonal Modes (POM), from experimental data or from numerical simulations, for subsequent use in a Galerkin projection that yields low-dimensional dynamical models. These functions are optimal in the sense that fewer POD modes are needed to account for the same amount of "signal energy" compared to any other orthogonal basis[5]. Thus, the POMs are a minimal set of output signals that can be used to identify the dominant eigenvalues of the *Poincaré* map jacobian matrix. After evaluating the time history associated with these signals, it is possible to extract the information regarding the dominant eigenvalues by means of standard system identification procedures. At the same time, the shapes of the modes associated to the eigenvalues of the *Poincaré* map jacobian matrix can be reconstructed, yielding a physical representation of the movement related to that frequency.

2 EIGENSOLUTIONS FOR INDEX THREE DAE SYSTEMS

The dynamic analysis of a complete multidisciplinary multibody model is addressed. The main methodologies used to investigate the dynamic behaviour of a complex system by multibody codes are based on the analysis of time histories resulting from the solution of Initial Value Problems (IVP). However, sometimes it may be necessary to synthesize the huge amount of information generated by the numerical simulation, to have a small set of charateristic figures which globally describe the system dynamics. For linear(–ized) systems these are the eigenvalues and the eigenvectors (or modes), which will be globally indicated as eigensolutions. Anyway, when the model is large, the computation of the eigenvalues over the whole spectrum may become a computationally demanding task, besides being mostly unnecessary. A method must be found that is capable of estimating and selecting only the dominant eigenvalues which characterize the phenomenon under investigation, while reducing as much as possible the computational burden.

The mathematical model of multibody systems, based on a redundant set formulation, is made of a large index three *Differential–Algebraic* system of *Equations* (DAE)[6], which contains, together with the usual differential equations that represent the dynamics of the mechanical degrees of freedom, the algebraic/differential equations, which describe the kinematic holonomic/nonholonomic constraints, and the equations that describe the dynamics of the multidisciplinary fields. The integration of this system requires a special treatment[6;7], owing to the singularity of the algebraic equations when the problem is treated as differential. By calling y the kinematic unknowns, z the momentum unknowns and λ the algebraic *Lagrange* multiplier unknowns, the system can be cast in the following general form

$$\begin{aligned} &\boldsymbol{M}(y,t)\dot{y} = z, \\ &\dot{z} = Q(y,\dot{y},t) - \boldsymbol{G}^T\lambda, \\ &\Psi(y,t) = 0. \end{aligned} \tag{1}$$

In these equations, $\boldsymbol{M}$ is a configuration dependent inertia matrix, Q are arbitrary external forces and couples, and $\boldsymbol{G} = \Psi_y$ is the jacobian of the holonomic constraints with respect to the kinematic unknowns. The final system is DAE of index three, meaning that three differentiations with respect to time are required to obtain $\dot{z}$ as a continuous function of (y,t)[6]. Nonholonomic constraints require a slightly different treatment and result in a lower index DAE system, whose solution is less critical. Numerous techniques have been proposed to solve this kind of problems[6;7]. All the cases presented in this paper are solved directly in the DAE form of Eq. (1), resorting to a fully implicit A/L-stable, second order accurate predictor-corrector integrator[1]. As a consequence, the equations are treated as a stiff system of differential equations, but the tangent jacobian matrix of Eq. (1) may not be the correct one to obtain the motion eigensolutions.

In fact, for index three DAE systems, a constraint represents a $2n - m$ dimensional manifold

$$\mathcal{M} = \{(y,z) \mid \Psi(y,t) = 0, \quad \boldsymbol{D}_y\Psi(y,t)\,\boldsymbol{M}^{-1}(y,t)\,z = 0\}$$

on which the solution must lie, where n is the dimension of the y vector, and m is the number of constraint equations $\Psi(y,t) = 0$. The dynamic behavior of the system represented by Eq. (1) will be locally dominated by the eigenvalues of the linearized vector field that lies in the tangent space of $\mathcal{M}$:

$$\begin{aligned} T_{(y,z)}\mathcal{M} = \{(v,w) \mid \Psi_y(y,t)v = 0, \quad &\Psi_{yy}(y,t)\,(M^{-1}(y,t)\,z)\,v \\ &+ \Psi_y(y,t)\left[(M^{-1}(y,t))_y\,z\,v + M^{-1}(y,t)\,w\right] = 0\}. \end{aligned} \tag{2}$$

Consequently, to obtain the eigensolutions the correct tangent matrix needs to be constructed; it must comply with the conditions of Eq. (2), which are not satisfied in general for the jacobian matrix of Eq. (1).

2.1 An example: rotating mass with offset

Consider the following example, where a rotating mass, representative of a rotating helicopter blade, of unit mass and pinned to a point which is offset from the rotating axis, is described, using cartesian coordinates and *Lagrange* multipliers:

$$\begin{aligned}
\dot{x} &= u \\
\dot{y} &= v \\
\dot{z} &= w \\
\dot{u} &= -2\left(x - x_h\right)\lambda \\
\dot{v} &= -2\left(y - y_h\right)\lambda \\
\dot{w} &= -2z\lambda \\
\left(x - x_h\right)^2 + \left(y - y_h\right)^2 + z^2 &= L^2
\end{aligned}$$

This index three DAE correctly describes the problem. If the pin point, which is offset, moves with law $\{x_h, y_h\}^T = h\{\cos\left(\Omega t\right), \sin\left(\Omega t\right)\}^T$, then the body, if appropriately excited, will oscillate with approximate in-plane and out-of-plane frequencies

$$\omega_\xi = \Omega\sqrt{\frac{h}{L}}, \qquad \omega_\beta = \Omega\sqrt{1 + \frac{h}{L}}$$

about the equilibrium trajectory:

$$\left\{\begin{array}{c} x \\ y \\ z \\ \lambda \end{array}\right\} = \left\{\begin{array}{c} \left(h + L\right)\cos\left(\Omega t\right) \\ \left(h + L\right)\sin\left(\Omega t\right) \\ 0 \\ \Omega^2\left(1 + h/L\right)/2 \end{array}\right\}.$$

However, if the generalized eigenvalues of the jacobian matrix of the linearized problem

$$\left[\begin{array}{ccccccc}
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
2\lambda & 0 & 0 & 0 & 0 & 0 & 2\left(x - x_h\right) \\
0 & 2\lambda & 0 & 0 & 0 & 0 & 2\left(y - y_h\right) \\
0 & 0 & 2\lambda & 0 & 0 & 0 & 2z \\
2\left(x - x_h\right) & 2\left(y - y_h\right) & 2z & 0 & 0 & 0 & 0
\end{array}\right]\left\{\begin{array}{c} \Delta x \\ \Delta y \\ \Delta z \\ \Delta u \\ \Delta v \\ \Delta w \\ \Delta\lambda \end{array}\right\}$$

$$+\left[\begin{array}{ccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]\left\{\begin{array}{c} \Delta\dot{x} \\ \Delta\dot{y} \\ \Delta\dot{z} \\ \Delta\dot{u} \\ \Delta\dot{v} \\ \Delta\dot{w} \\ \Delta\dot{\lambda} \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\}$$

are considered, one obtains, as expected: one minus-infinite eigenvalue, that is related to the constraint unknown; two null eigenvalues, related to the axial displacement and momentum modes; then, two eigenvalues of the form

$$\omega = \pm\Omega\sqrt{1 + \frac{h}{L}}$$

with multiplicity 2, related to the in-plane and out-of plane displacements, are (unexpectedly?) found. The tangent matrix of the DAE system does not distinguish between the two transverse motions. In fact, it contains no information on the motion of the constraint point, so the two transverse directions are equivalent, and generate the same small perturbation motion, characteristic of a flapping (i.e. out-of-plane) displacement. The constraint reaction force, in both cases, is related to an *axial* rotation motion, while, for the in-plane case, it should be related to a *central* rotation motion.

As a consequence, the jacobian matrix of the DAE problem, without even considering how difficult it may be to extract significant information from large size problems, does not contain the characteristic parameters of the motion.

3 TRANSITION MATRIX APPROACH

A different approach may be followed to find the eigensolutions, starting from the results of numerical simulations. This allows to overcome all the hurdles related to the DAE index 3 formulation. The transition matrix allows to write the following expression

$$x^{(k+1)} = \Phi(h, k)\, x^{(k)}, \tag{3}$$

where the index k is related to samples one integration time step h apart. In the above written recursive equation the matrix Φ must be evaluated starting from appropriate vectors $x^{(k)}$, which are computed numerically by integrating Eq. (1) after applying a suitable initial perturbation. The perturbation strength must guarantee an appropriate linearization, while keeping the solution within the attraction basin of the equilibrium orbit under investigation (which can be either a stationary point or a periodic orbit). Different approaches for the determination of the eigenvalues of Φ are related to different interpretations of Eq. (3) (see Quaranta *et al.*[4]). In any case, the eigensolutions are evaluated in the discrete time domain; then they must be trasformed in the continuous time domain through the relation:

$$\lambda = \frac{1}{h}\ln\Lambda = \frac{1}{h}\ln|\Lambda| + \frac{i}{h}\left[\arctan\frac{\mathrm{Im}\,\Lambda}{\mathrm{Re}\,\Lambda} + 2\pi j\right], \tag{4}$$

where "ln" indicates the principal natural logarithm, and j is an integer.

Bauchau and Nikishkov[8] propose an *implicit matrix method* that exploits the properties of the *Arnoldi*'s algorithm, a subspace method, which needs only a matrix-vector multiplication to extract the highest modulus eigenvalues of large and sparse matrices. Therefore, the eigenvalues of the transition matrix Φ can be obtained by performing a computation of the response after a certain time interval to an initial condition vector chosen by the algorithm. The method is applied by *Bauchau and Nikishkov*[8] to periodic stability analyses. Anyway, if a sufficiently small time interval is chosen, the eigenvalues of the linearized system can be evaluated near an equilibrium condition. Unfortunately, the evaluation of the eigenvalues directly from the degrees of freedom of index three DAE systems will result in $2m$ spurious eigenvalues that will not give any useful insight into the dynamics of the mechanism. Usually the m eigenvalues associated to the *Lagrange* multipliers assume a minus infinite value, to indicate that the constraint equations are instantaneously satisfied (infinitely fast dynamics). The eigenvalues associated with the redundant coordinates, instead, assume a zero value, since the constrained degrees of freedom have no autonomous dynamics. Moving from the continuous to the discrete time domain, the zero-valued eigenvalues associated with the constraints are morphed in eigenvalues on the unit circle in the space of the discrete dynamical system, by means of the inverse of Eq. (4). Consequently, the first m eigenvalues of a stable problem found by *Arnoldi*'s method are all related to the constraints, so they do not give any useful insight into the dynamics of the investigated system[4]. In order to simplify the computation, a different technique to select the dominant dynamics must be devised.

4 PROPER ORTHOGONAL DECOMPOSITION

A way to select a reduced set of signals, synthesizing the information related to the dominant transient, must be devised to be able to apply any method for the evaluation of stability properties. These signals can be selected by resorting to a technique capable of extracting spatial coherence in an oscillating system, when the time history of its state variables is known from either a numerical simulations or from the output of several sensors measured in real-life experiments. This method is represented by the Proper Orthogonal Decomposition (POD), used for the analysis of multidimensional data. It provides a way to find the best approximating subspace to a given set of data in a least-square sense. The POD allows one to obtain a modal decomposition that is only data dependent and does not assume any prior knowledge of the system. Consequently, it perfectly fits the analysis needs of multidisciplinary models. A reduced order model of a dynamical system can be obtained by simply using a Galerkin projection procedure, where the POD modes are used as basis functions. The same ideas can be applied here to the generation of a small subset of significant signals to be used for the identification of the leading eigenvalues, characterizing both system behavior and stability. For the analysis of mechanical systems, it should be further noticed that these base POMs have an interesting interpretation, since they can be viewed as the result of a least-square error optimization for a linear representation of the nonlinear normal modes[9]. By means of the POD all the eigenvalues associated with the constraint will be excluded since no work, and consequently no energy, is associated with them.

4.1 Synthesis by means of POD

Consider a system where all the N state variables are measured at n time steps. Their time averages are usually subtracted from the signals and data are arranged in a $N \times n$ matrix

$$X = [x^{(1)}, x^{(2)}, \ldots, x^{(n)}]. \tag{5}$$

An approximation of the system dynamics is obtained by projecting the original N-dimensional state space onto an m-dimensional subspace $\mathcal{S}$. The main purpose of POD is to find a projection operator Q mapping $\mathbb{R}^N$ onto $\mathcal{S}$, which minimizes the Euclidean distance of the sampled points from the m-dimensional hyperplane

$$H(Q) = \sum_{i=1}^{n} \|x^{(i)} - Qx^{(i)}\|. \tag{6}$$

It can be shown[5] that, being $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ the eigenvalues of the data correlation matrix $E = XX^T$, the optimal m-dimensional projection operator is represented by the $m \times N$ matrix whose rows are the first m eigenvectors of E. To obtain these eigenvectors, assume to compute the singular value decomposition (SVD) of matrix X^T, i.e.,

$$X = U\Sigma V^T, \tag{7}$$

where Σ is the diagonal matrix of the singular values σ_i, and U, V are unary rectangular matrices. By sorting the singular values σ_i of X in descending order, it can be shown that the matrix $X_m^T = U_m \Sigma_m V_m^T$ is the closest rank m matrix to X in the *Frobenius* norm, where U_m and V_m are the rectangular matrices obtained by retaining the first m columns, and Σ_m is the $m \times m$ principal minor of Σ. The correlation matrix E can be expressed as:

$$E = XX^T = U\Sigma V^T V \Sigma^T U^T = U\Sigma\Sigma^T U^T. \tag{8}$$

Eq. (8) implies that U is the eigenvector matrix of E, and the corresponding eigenvalues are the squares of the singular values. To choose the dimension m of the approximate subspace that will contain all the significant information, the singular values must be considered, since their value expresses the "signal energy" related to the associated POM. These values drop to a constant plateau, usually called "noise floor", which characterizes the modes that do not contain any significant information[10]. In fact, the use of some form of SVD as a tool to compute the order of a model is a common practice in system identification as well.

If $N \gg n$, then it is more efficient to use the so-called *method of snapshots* in POD literature. It consists in first computing the matrix V as the eigenvectors matrix of $X^T X$. Once V is known, since

$$XV = U\Sigma,$$

it is clear that the norm of the first m rows of $U\Sigma$ are the singular values, and that these rows, after normalization, are the POMs. Once the POM forms are known, one can obtain their time histories which may be subsequently identified by means of the classical AR algorithm based on Eq. (3)[11]. To obtain a set of signals of comparable amplitude, which will greatly improve the identification phase, it is important to use a set of "balanced" POMs, such as $U\sqrt{\Sigma}$. To obtain a better identification of the resulting signal, it may be useful to employ a second order AR identification algorithm

$$X^{(k+1)} = \Phi_1 X^{(k)} + \Phi_2 X^{(k-1)}, \tag{9}$$

since it is known that mechanical systems are characterized by second order dynamics.

4.2 Selection of perturbations

When dealing with systems with a large number of degrees of freedom, the choice of an effective set of perturbations may not be a trivial task. If external force impulses are used, it cannot be safely asserted, in general, that any set of initial impulses will excite all the modes of the system. Often a prior knowledge of the physical phenomena under investigation will help in choosing the perturbation type and level. An advantage of the presented method to find the eigenspaces of the system under investigation is related to the possibility of choosing the frequency range to be swept. Anyway, a great care must be used in shaping the numeric impulse to obtain the desired results. In fact, usually a numeric impulse will transfer its energy mainly to the higher frequency segment of the spectrum.

In our extensive experiences, partly reported in the following, no significant problems were encountered when selecting suitable perturbations. The quality of each identified mode can be assessed using a "figure of merit", expressed as

$$F_i = \sum_j |\varphi_{ij}| \sigma_j, \tag{10}$$

where φ_{ij} is the j-th component of the i-th eigenvector of the POD state matrix. If F_i is low, the mode is not composed by the dominating POMs, so probably the selected signal may not be suitable for this specific vibration mode.

Anyway, a more general excitation method can be obtained by using white noises inputs $w(t)$, such that $E[w(t)] = 0$ and $E[w(t)w(\tau)^T] = \nu\delta(t-\tau)$. The system is assumed to be stable and time invariant, and all processes are assumed to be gaussian, which is true for the cases under investigation, since we are mostly interested in the linearization about an equilibrium orbit. It can be shown that the resulting correlation matrix E is the solution of the following Lyapunov equation[12]

$$AE + EA^T + B\nu B^T = 0, \tag{11}$$

where A is the usual state-space system matrix, and B is the input matrix which express the connection between the inputs and the internal states. The same equation is used to compute to the controllability grammian[13], which contains all the dynamics reachable from the selected inputs, so E is also the controllability grammian. Using a band-limited white noise, whose spectrum is flat over a limited frequency range, an approximation of the dominant eigenvalues of the controllability grammian can be obtained in the desired frequency range. The resulting reduced order model, which will be used in the subsequent identification phase, is thus correlated to what can be obtained by the Balanced Truncation, a classical system reduction technique[13], without resorting to the explicit knowledge of the state-space system matrices A and B. In this case Eq. (3) must be slightly modified to account for the (known) input signals:

$$X^{(k+1)} = \Phi(h, k)\ X^{(k)} + a^{(k)}, \tag{12}$$

where $a^{(k)}$ represents the stochastic part which has no dependence on the past states and is independent also of $X^{(k)}$, $X^{(k-1)}$, ..., $X^{(k-n)}$. This technique will be also useful in all the situations where a persistent perturbation may help, such as with highly damped systems.

5 ROTOR MODEL ANALYSIS

The technique that was presented in the previous sections is applied to the aeroservoelastic analysis of the dynamical properties of a helicopter tailrotor, including the hydraulic actuator and portions of the control system. The problem is modeled by means of the multibody/multidisciplinary analysis software MBDyn[1]. The model is representative of a generic helicopter tailrotor, and it was validated by comparisons with results from comprehensive rotorcraft analysis codes during a previous research work[14]. In the above mentioned work, the focus was on the servoaeroelastic analysis of the tailrotor to identify the aeroelastic transfer function of the rotor for detailed servoactuator design and performance analysis. Since the tailrotor is controlled by means of the collective pitch only, in that case a single-input, single-output model was used. Now, a completely different problem is considered; the entire rotor must be identified, so all the degrees of freedom are taken into account, and different inputs need to be used. If no model reduction is performed, on one hand the problem would be very large; on the other hand, a lot of information of little use would be found.

5.1 Tailrotor multibody model

The multibody model of the tailrotor is made of a rotating hub, carrying four deformable blades, hinged at the root, which is offset from the hub, by elastomeric bearings. The lead-lag motion of each blade is also constrained by an elastomeric damper. The blades are modeled by an original finite volume beam formulation[15], with lumped inertia. The aerodynamics of the rotor blades is very simple, based on strip theory with table lookup of the airfoil static coefficients, empirical unsteady aerodynamics and dynamic stall correction according to[16]; rotor inflow is accounted for. The hub is attached to a deformable mast, whose angular velocity is imposed by means of kinematic constraints. The mast is attached to a gearbox, which is grounded. The control system is made of a rod that is placed inside the mast, which controls the axial displacement of a four-leg spider. The spider legs are lagged 45º behind the respective blades, and control the collective pitch by means of deformable pitch links. As usual in tailrotors, there is significant positive pitch flap coupling. In some of the analyses, the pitch rod displacement is controlled by means of a servohydraulic actuator, which in turn is controlled by a hydraulic circuit. This model was used in[14] to investigate the possibility to synthesize aeroelastic trasfer functions for the design and analysis of servoactuators, and to directly assess the hydraulic system designs in an integrated nonlinear analysis by means of the multidisciplinary capability of the multibody software.

5.2 Tailrotor analysis description

Different types of analysis are used for different purposes: transient analysis, model performance validation, mechanical validation, model synthesis.

a) The transient analysis is peculiar to the proposed multibody analysis approach, and all the other analyses are based on interpreting or especially crafting transient analyses.

b) The model performance validation is obtained by computing hover and forward flight trim points. While the former case results in basically steady solutions, due to the axial symmetry of the problem, the latter results in periodical solutions. Convergence in both cases is achieved by running as many rotor periods as required to dampen out the transient response; usually, this does not require more than three-four revolutions.

c) The model mechanical validation is achieved by performing POD analysis on transient results. It is the main focus of this paper, and its capabilities and results will be shown in the results section.

d) The model synthesis is obtained by identifying a reduced order model from the transient response, either directly, e.g. by assuming an input/output configuration as in [14] for the tailrotor control system response to the input of the actuator, or by means of modal condensation, as obtained with POD reduction prior to system identification. In either case, the model behavior is computed from a transient analysis with appropriate input.

To properly identify the mechanical properties of the system, regardless of its nature (e.g. whether it is purely mechanical, aeroelastic or servoaeroelastic), there are two main possibilities: a free or a forced system analysis.

If the system is naturally forced, and significantly if it is periodical (or periodically forced, which is equivalent), then a simple transient analysis, without any extra forcing term, will yield significant information about the stability properties of the motion. In fact, if the system is stable, the transient resulting from an untrimmed initial value condition will converge to a steady condition (regardless of whether it is periodical or not) with a rate that depends on the stability properties of the system, or at least on the stability properties of that specific solution, if the system is nonlinear. By perturbing the steady solution, and exploring the phase space in its vicinity, one can assess its stability in an exaustive manner; however, in practical cases, the perturbations introduced by the untrimmed initial values suffice.

The typical POD analysis to identify the eigenvalues and eigenvectors of the system is performed *in vacuo* first, with separate collective, cyclic and reactionless excitation, to exploit significant system symmetries. This is common practice in helicopter rotor analysis, and, even within the analysis framework that is presented here, it helps separating different mode shapes with very close, possibly coincident, frequencies.

The *in vacuo* analysis is necessary to validate the model first, by comparing with available modal analysis from finite elements or other analysis tools, but also to provide a knowledge of the peculiarities of the problem. The resulting frequencies and mode shapes help suggesting how to excite the problem and what to look for when analyzing the model with aerodynamic forces. In fact, when dealing with purely mechanical models, high damping resulting from system identification indicates a poor measure, which can be related to poor or inappropriate excitation. While the figure of merit of Eq. (10) can be of help, it does not suffice, because tricky systems can show small figures relating to correct modes, which would be lost, or the opposite, which is even worse.

5.3 Results

Tables 1, 2 show the normalized frequency, $\text{imag}(\lambda)/\Omega$, and the damping ratio, defined as $-100\text{real}(\lambda)/|\lambda|$, for the rotor *in vacuo* and in forward flight at a high advance ratio ($\mu \cong 0.40$). In both cases, colored noise forces and couples at blade tips, both in flap and lag directions and about the twist axis, have been applied to obtain a wide spectrum excitation all over the blades. In this sense, the numerical experiment is not practically obtainable by usual rotorcraft aeroelastic testing procedures.

A different excitation procedure has been tried, consisting in applying the broadband input to the collective control. This produces, as expected, a very good identification of the collective torsional modes, and, for the model with air, a less than optimal identification of the flap modes, which interact with blade torsion by means of the aerodynamic forces. This technique is based on a physical ground and can be used also in real life experiments.

Table 1 shows that while high frequency modes with limited damping can be easily detected, the low frequency modes may pose some problems. In detail, the rigid lead-lag motion was not found with cyclic and reactionless excitation modes, because it is highly damped by the lead-lag damper. As a consequence, the broadband excitation was not able to force its response out of the noise floor. The collective response was measured, although with limited accuracy, because the collective mode is excited also by the wind-up transient. The measure is very dependent on the identification parameters, so it should be considered an indication of a poorly excited mode, and discarded. This was clear also from the value of the figure of merit resulting from Eq. (10). In an analogous manner, the rigid

Mode	Collective		Cyclic		Reactionless	
	1/rev	%	1/rev	%	1/rev	%
Rigid Lead-Lag	0.60	74.94	n.a.		n.a.	
Rigid Flap	1.11	1.42	n.a.		n.a.	
1^{st} Deformable Flap	3.14	0.62	3.16	6.40	3.12	4.59
1^{st} Torsion/Control	4.34	0.37	5.04	2.65	5.07	1.00
2^{nd} Deformable Flap	6.82	0.26	6.90	0.56	6.90	0.26
2^{nd} Torsion/Control	7.85	0.18	7.79	2.96	n.a.	

Table 1: Rotor modes *in vacuo*

flapping motion, which is close to 1/rev, was not captured with cyclic and reactionless excitation modes, possibly because it does not emerge from the dominating collective response related to the wind-up excitation. Finally, the damping on well-measured modes must be very limited, because the amount of structural damping, besides the lead-lag damper, was minimal. As a consequence, an appreciably damped mode in general is indicative of a poor identification. Figure 1 shows the mode shape of the first deformable flap mode *in vacuo*, to highlight how clean a mode separation can be obtained. Figure 2 shows the mode shape of the first torsion/control mode *in vacuo*; notice how the collective mode shows an appreciable participation of the control, indicated by the finite rotation at the blade hinge.

Table 2 again shows that the rigid blade modes cannot be accurately identified. The lead-lag modes are not measured at all; the identification captures some reasonable lead-lag shapes with frequency below 1/rev, but they barely pass the noise floor, so they have not been reported. The higher frequency modes are measured with good accuracy with all the excitation modes that were considered. They show the expected trends with respect to the *in vacuo* ones: the frequency of the twist/torsion modes is reduced by the aerodynamic forces, in fact the aerodynamic center of the airfoils is slightly ahead of the feathering axis; the rigid flap mode (although clearly measured only with reactionless excitation, while *in vacuo* it was measured in the collective case) is increased by the positive pitch-flap coupling. Figure 3 shows that the first deformable flap mode cannot be clearly separated any longer in the collective case, which is now a combination of all the shapes. On the contrary, Figure 4 shows that the first torsion/control mode is still well separable.

Finally, note how, both with and without aerodynamic forces, the collective modes do not differ much in the collective, cyclic and reactionless case, except for the first torsion/control mode. This is related to the configuration of the control system that is peculiar of tailrotors: since only the collective control is present, the first collective mode implies an appreciable participation of the control flexibility, while the cyclic and the reactionless modes imply no appreciable control deformation, and thus show a stiffer behavior.

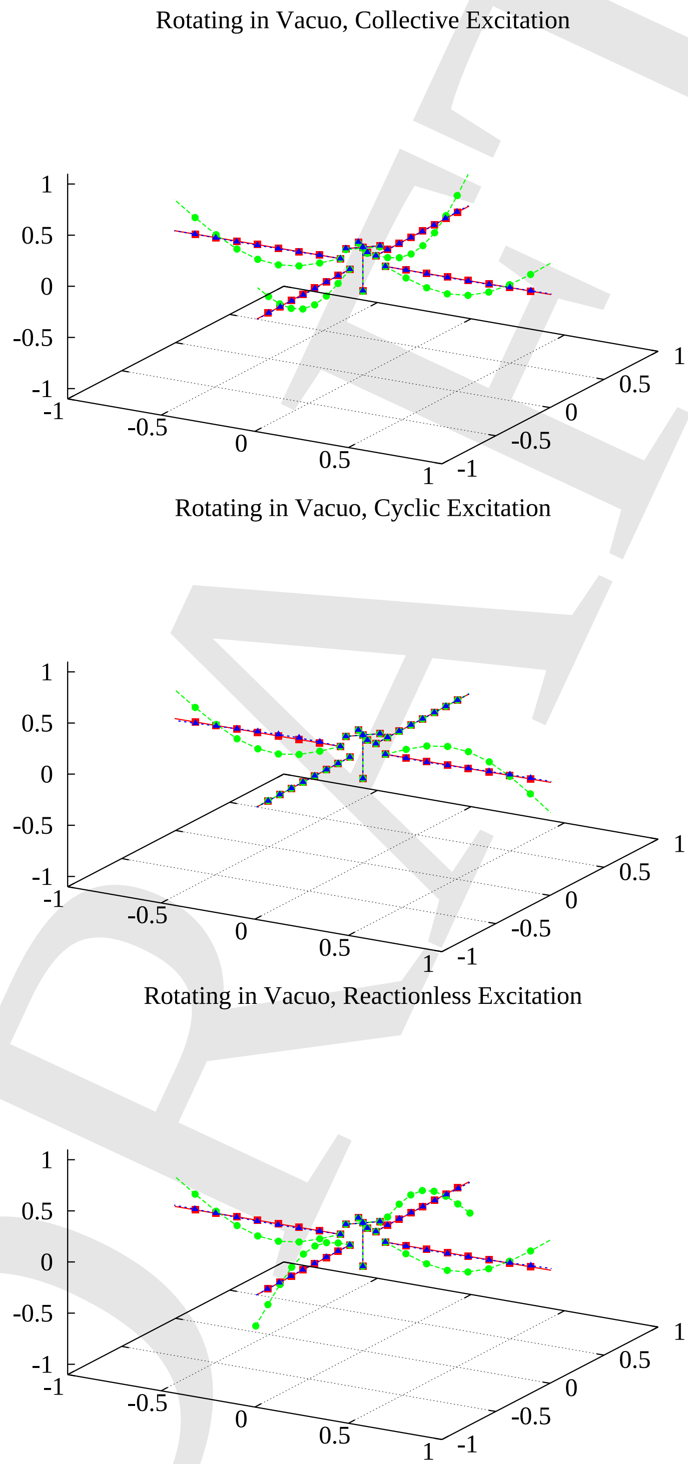

Figure 1: 1st deformable flap mode *in vacuo*

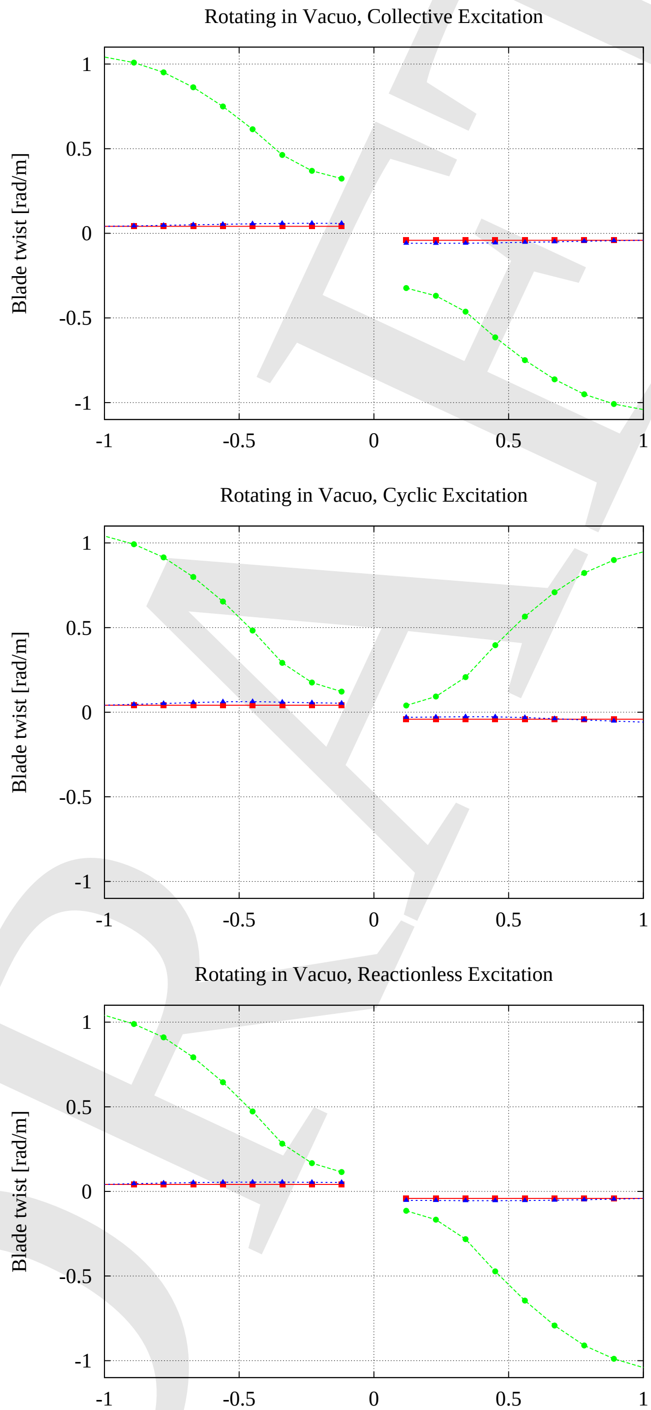

Figure 2: 1st torsion/control mode *in vacuo*

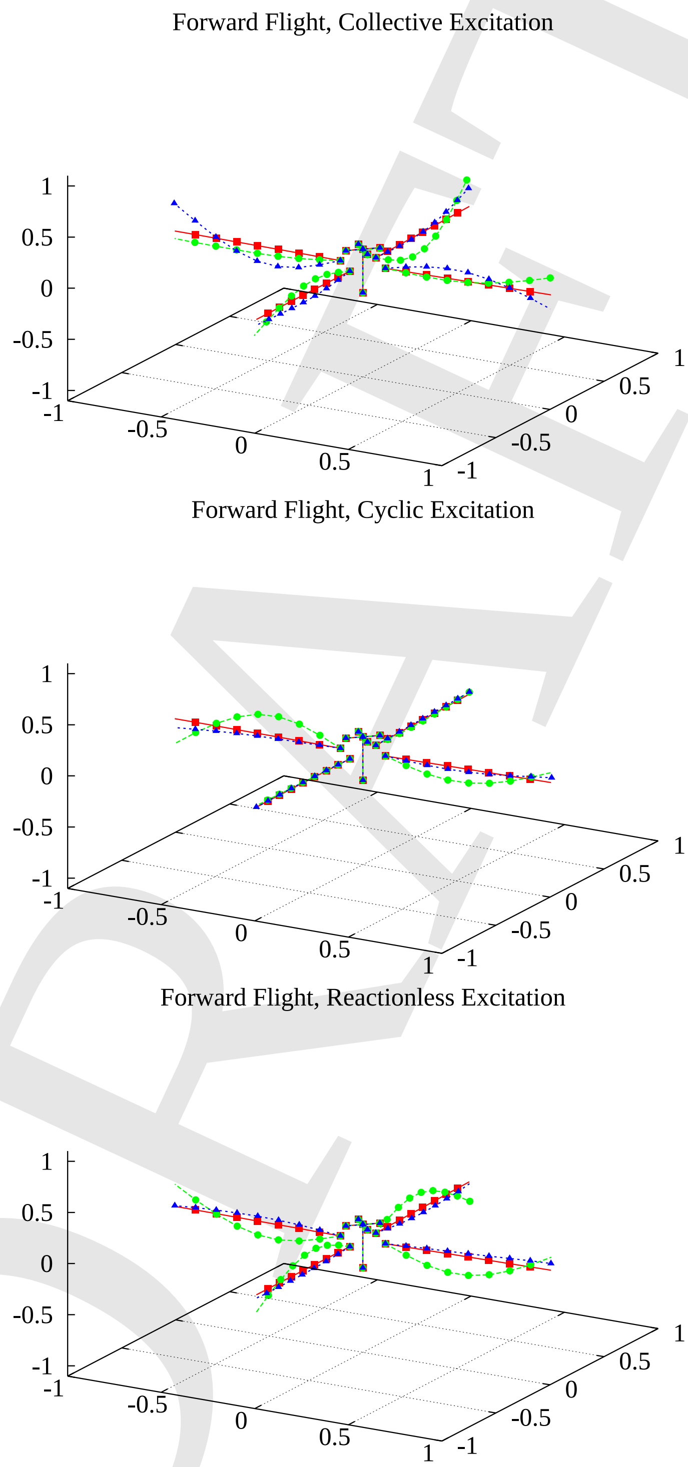

Figure 3: 1st deformable flap mode in air

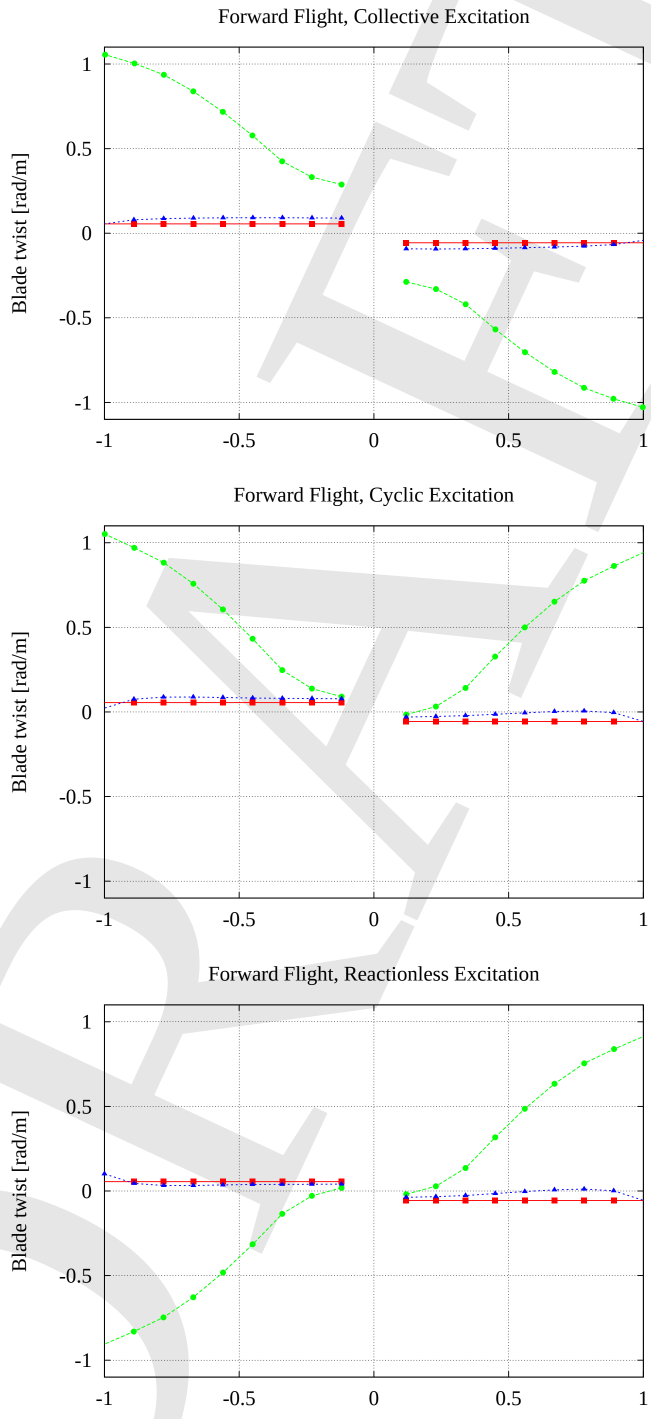

Figure 4: 1st torsion/control mode in air

Mode	Collective		Cyclic		Reactionless	
	1/rev	%	1/rev	%	1/rev	%
Rigid Lead-Lag	n.a.		n.a.		n.a.	
Rigid Flap	n.a.		n.a.		1.23	20.63
1st Deformable Flap	3.09	1.29	3.03	6.92	2.97	5.59
1st Torsion/Control	4.10	0.91	4.46	3.55	4.53	2.77
2nd Deformable Flap	6.73	1.22	6.77	1.37	6.79	1.12
2nd Torsion/Control	7.76	0.56	7.11	5.97	7.13	21.57

Table 2: Rotor modes in forward flight

6 CONCLUSIONS

An application of the Proper Orthogonal Decomposition to the characterization of helicopter rotors by means of multibody analysis has been presented. It represents a valid alternative to approaches based on the computation of the transition matrix, which may suffer from some limitations in identifying stable eigensolutions. This work has been partially supported by Agusta S.p.A., a company of AgustaWestland, under Research Contract NM-000/1505.

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