

DYNAMIC CHARACTERIZATION AND STABILITY OF A LARGE SIZE MULTIBODY TILTROTOR MODEL BY POD ANALYSIS

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ABSTRACT

A great level of flexibility and accuracy can be gained by employing a multibody approach as the modeling strategy for servo-aeroelastic analyses of rotating wing aircraft. However, the resulting models are often large, in terms of degrees of freedom, and fully nonlinear, making the dynamic characterization phase cumbersome and computationally demanding. Moreover, non-minimal set formulations such as the one used here to express the equations of motion may cause the rise of spurious eigenvalues. This paper presents the application to a tiltrotor model of a technique which associates the POD (Proper Orthogonal Decomposition) methodology, to reduce the impact of the model large dimensions, with standard dynamic system identification methods, to efficiently circumvent the complexity usually associated with the development phase of flexible multibody models. The same technique is also employed to assess the periodic motion aeroelastic stability of the tiltrotor.

INTRODUCTION

The correct analysis of the dynamic behavior of complex systems, such as tiltrotors, requires sophisticated analyses, which can be effectively carried out by means of general purpose modeling codes based on multibody/multidisciplinary methodologies. The need to account for large rotations and displacements naturally leads to the use of multibody analysis codes. In fact, a great impulse to the development of these instruments originated from the needs of (flexible) rotor dynamics and aeroelasticity. The multibody approach allows to use modular models to build a complete system with the required detail for each subpart. As

a result, the same model (or common components) can be used for the different analyses that are required at each stage of development. One of the most significant features is that kinematics have an exact formulation within the chosen idealization of the real system.

By means of these codes the designer can progress from simple rigid body schemes up to fully detailed flexible models analyses, including accurate and fully nonlinear constraints, deformable elements, servohydraulic circuits, aerodynamic forces and control system components [1]. The size of the resulting model, boosted by the increasing demand for details in deformable bodies modeling, and accuracy in the aerodynamic loads predictions, may soon become very large.

The complexity of the whole model is dictated by the complexity of the problems under investigation and by the advantages of a generic approach; however, to make this large models useful in the design and analysis phases, a methodology capable of synthesizing a relatively small set of significant results in each condition is required. This need is particularly strong when stability analyses are addressed.

Another level of entanglement is added by the redundant set formulation used to build the equations of motion for deformable multibody systems, in which the constraints between inertial bodies are explicitly expressed [2]. In fact, the direct analysis of the Jacobian matrix of the resulting index 3 *Differential Algebraic* system of *Equations* (DAE) [3] for the computation of the eigenvalues and eigenvectors characterizing the dynamic behavior of the system under investigation, may lead to incorrect spurious solutions.

This paper presents an effective method to overcome these hurdles based on *Proper Orthogonal Decomposition* (POD), also known as *Karhunen-Loève Decomposition*. For a brief histori-

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Figure 1. ERICA EUROPEAN ADVANCED TILTROTOR; BY COURTESY OF AGUSTA.

cal review of the origins of POD the reader should refer to [4]. POD is a powerful and elegant method of data analysis aimed at obtaining a low-dimensional approximate description of high-dimensional processes. There are many applications of POD. It has been extensively used in fluid mechanics, to study turbulent flows and to reduce the degrees of freedom of numerical models [5], or in the reduction of complex viscous transonic aerodynamic fields inside a turbomachinery [6]. Recently, it has been applied to dynamic studies of structural vibrations [7, 8], and damage detection [9]. Data analysis using POD is conducted to extract a set of basis functions, called Proper Orthogonal Modes (POM), from experimental data or from numerical simulations, for subsequent use in a Galerkin projection that yields low-dimensional dynamical models. These functions are optimal in the sense that fewer POD modes are needed to account for the same amount of "signal energy" compared to any other orthogonal basis [5]. Thus, the POMs are a minimal set of output signals that can be used to identify the dominant eigenvalues of the Jacobian matrix. After evaluating the time history associated with these signals, it is possible to extract the information regarding the dominant eigenvalues by means of standard system identification procedures.

This technique will be here applied to the aeroelastic stability analysis of ERICA, the European advanced tiltrotor project [10] shown in Figure 1, in partial fulfilment of the requirements of the ADYN EU Project. This new tiltrotor will include several innovative concepts that differentiate it from existing configurations, such as a structural connection of the tilting mechanism, hingeless hub-blade connections and hingeless gimbal mechanism, reduced diameter rotors and tiltable wing. All this features yield a complex layout that can be adequately analyzed within the multibody framework. All the presented analyses are performed by means of MBDyn, an open-source multi-

body/multidisciplinary code, developed at the *Dipartimento di Ingegneria Aerospaziale* of University *Politecnico di Milano* (see <http://www.aero.polimi.it/~mbdyn>).

After a brief description of the problems associated with the eigenanalysis of non-minimal set multibody models, the POD method and its application in the dynamic characterization phases are presented. A detailed description of the application to the tiltrotor model follows. The quality of the proposed method is shown by matching its results with those obtained from FEM codes; at the same time the versatility of the approach is highlighted. A very useful feature for the investigation of rotating machines is the possibility, easily implemented within the POD framework, to run analyses in fixed or rotating reference frames. A final section is dedicated to the application of the same technique to stability analyses.

MULTIBODY SYSTEMS EIGENVALUES

The large scale models that are investigated in this work fall into the class of multidisciplinary multibody systems [2]. They are represented by special DAE systems. To clarify the peculiarities of these models, a brief presentation of the basic formulation is needed.

The motion of a system of rigid/deformable bodies can be described using the principles of classical mechanics. They may lead to a redundant coordinate set approach, in which the constraints between inertial bodies are explicitly expressed, or to a minimal coordinate set approach where all the constraints are eliminated. The numerical reduction to minimal set equations usually results in methods which are computationally less effective if complicated mechanisms are investigated. The redundant approach instead can be used to easily and automatically generate the equations modeling almost any complex system by means of the multibody formalism. A multibody system may be defined as a collection of bodies in arbitrary motion with respect to each other. These bodies can be connected by algebraic/differential constraints and flexible elements; they can also be subjected to interactional forces that represent other multidisciplinary elements.

The basic equations of motion for the constrained bodies can be described referring to the Lagrangian multipliers technique to introduce algebraic constraints. The dynamics of the bodies are written in form of a fully implicit first-order differential system by coupling the definition of the momentum, β , and of the momenta moment, γ , of each body to the force and moment equilibrium equations [1]. The resulting set of equations for each body

is

$$\begin{aligned}
\beta &= m\dot{x} + \omega \times S, \\
\gamma &= S \times \dot{x} + J\omega, \\
\dot{\beta} &= F(x, R, \dot{x}, \omega, a, \dot{a}, \dots, t) + V_F, \\
\dot{\gamma} + \dot{x} \times \beta &= C(x, R, \dot{x}, \omega, a, \dot{a}, \dots, t) + V_C,
\end{aligned} \tag{1}$$

where the forces F and couples C may depend on the configuration, represented by the position vector x , the orientation matrix R and the translational and angular velocities \dot{x} and ω , and on the internal states a , that can be generally associated to the multidisciplinary fields that are simultaneously considered. The forces V_F and V_C are related to the Lagrangian multipliers and represent the actual reaction forces and couples generated by the constraints. Kinematic constraints, both holonomic and nonholonomic, are added as algebraic/differential equations:

$$\begin{aligned}
\Psi_h(x, R, \dots, t) &= 0, \\
\Psi_{nh}(x, R, \dot{x}, \omega, \dots, t) &= 0,
\end{aligned} \tag{2}$$

The resulting system of nonlinear DAE, obtained joining Equations (1) and (2), requires a special treatment [3, 11] due to the singularity of the algebraic equations, if the problem is treated as differential. By calling y the kinematic unknowns, z the momentum unknowns and μ the algebraic Lagrangian multipliers unknowns, the system can be cast in the following general form

$$\begin{aligned}
M(y, t)\dot{y} &= z, \\
\dot{z} &= Q(y, \dot{y}, t) - G^T \mu, \\
\Psi(y, t) &= 0.
\end{aligned} \tag{3}$$

In these equations, M is a configuration dependent inertia matrix, Q are arbitrary external forces and couples and $G = \Psi_y$ is the Jacobian of the holonomic constraints with respect to the kinematic unknowns (nonholonomic constraint require a slightly different treatment not illustrated here for simplicity). The final system is DAE of index three, meaning that three differentiations with respect to time are required to obtain \dot{z} as a continuous function of (y, t) [3]. Numerous techniques have been proposed to solve this kind of problems [3]; all the examples presented in the following are solved directly in DAE form resorting to a fully implicit A/L-stable, at least second order accurate predictor-corrector integrator [1, 12].

For index three DAE systems the constraints represent a $2n - m$ dimensional manifold

$$\mathcal{M} = \{(y, z) \mid \Psi(y) = 0, \quad \Psi_y(y)M^{-1}(y)z = 0\}, \tag{4}$$

on which the solutions must lie, where n is the dimension of the y vector, and m is the number of constraint equations $\Psi(y) = 0$. The dynamic behavior of the system represented by Eq. (3) will be locally dominated by the eigenvalues of the linearized vector field that lies in the tangent space of \mathcal{M}

$$\begin{aligned}
T_{(y,z)}\mathcal{M} = \{(v, w) \mid \Psi_y(y)v = 0, \quad \Psi_{yy}(y)(M^{-1}(y, t)z)v \\
+ \Psi_y(y)[(M^{-1}(y))_y z v + M^{-1}(y, t)w] = 0\}. \tag{5}
\end{aligned}$$

To obtain this information we should ideally compute the eigenvalues of the problem stated in terms of the minimal coordinate set, which represents the system of differential equations projected on the manifold \mathcal{M} . In any case, the evaluation of the eigenvalues directly on the DAE systems will result in $2m$ spurious eigenvalues that will not give any useful insight into the dynamics of the mechanism. Usually the m eigenvalues associated to the Lagrangian multipliers assume a minus infinite value, to indicate that the constraint equations are instantaneously satisfied (infinitely fast dynamics). The eigenvalues associated with the redundant coordinates instead, assume a zero value, since the constrained degrees of freedom have no autonomous dynamics. The correctness of the other $2n - m$ eigenvalues computed directly on the Jacobian matrix of the DAE system of Eq. (3) depends on the techniques actually implemented for the system solution. A simple calculation of the eigenvalues made directly on the Jacobian matrix obtained using the DAE formulation of Eq. (3) will result in a wrong eigensolutions, since, in general, it does not belong to the tangent space of the manifold \mathcal{M} .

Furthermore, since multibody multidisciplinary models are large, a method is needed to select only the global dynamics which dominate the system behavior in each specific configuration, without computing all the eigenvalues. In fact many of them can be affected by approximations related to the problem discretization.

EXTRACTING DOMINANT TRANSIENT BY SIMULATION

A different approach may be followed to find the eigensolutions, which circumvents all the problems related to the DAE index 3 formulation. A numerical simulation can be viewed as a “numerical experiment”, whose results may be analyzed by techniques that are suitable for experimental data as well.

For our purpose, a multibody system not subject to external input can be seen as an autonomous N -dimensional nonlinear dynamic system

$$\dot{x} = f(x). \tag{6}$$

The transition matrix allows one to write the following expres-

sion

$$x^{(k+1)} = \Phi(h, k) x^{(k)}, \quad (7)$$

where the index k is related to samples one integration time step h apart. In the above written recursive equation the matrix Φ must be evaluated starting from appropriate vectors $x^{(k)}$, which are computed numerically by integrating Equation (6) after applying a suitable initial perturbation. The perturbation strength must guarantee an appropriate linearization while keeping the solution within the attraction basin of the equilibrium orbit under investigation (which can be either a stationary point or a periodic orbit).

Different approaches for the determination of the eigenvalues of Φ are related to different interpretations of Equation (7). On one hand, it may be viewed as an auto-regressive (AR) discrete time model

$$X^{(k+1)} = \Phi(h, k) X^{(k)}, \quad (8)$$

where

$$\begin{aligned} X^{(k+1)} &= [\dots x^{(k+1)} x^{(k)} \dots], \\ X^{(k)} &= [\dots x^{(k)} x^{(k-1)} \dots], \end{aligned}$$

which is nothing but a generalization of the least square linear fitting of the system response to an initial perturbation. Using this technique, an approximation of the complete transition matrix is obtained, from which can be attained the full set of eigenvalues characterizing the discrete problem of Eq. (8). The eigenvalues λ of the linearized system matrix of the continuous time problem expressed in Eq. (6), and that of the transition matrix $\Phi(h, k)$, Λ , are related by

$$\lambda = \frac{1}{h} \ln \Lambda = \frac{1}{h} \ln |\Lambda| + \frac{i}{h} \left[\arctan \frac{\text{Im} \Lambda}{\text{Re} \Lambda} + 2\pi j \right], \quad (9)$$

where “ln” indicates the principal natural logarithm, and $j = 0, \pm 1, \pm 2, \dots$. Thus, there is an indeterminacy in the imaginary part of the continuous time eigenvalue of an integer multiple of $2\pi/h$, which represents the precision of the computed eigenvalues. In this way the analysis is independent from the formulation used to integrate the equation. However, the problem of the large dimension and of the elimination of the spurious eigenvalues related to the constraints has not been solved yet.

On the other hand, Eq. (8) can also be viewed as a power iteration, in which case an appropriate measure of the components ratio $x^{(k+1)}/x^{(k)}$, e. g. $x^{(k+1)} \cdot x^{(k)} / \|x^{(k)}\|^2$ is known to converge to the largest eigenvalue of $\Phi(h, k)$, if the initial vector is

not defective in the direction of the related eigenvector. Anyway, to obtain a complete dynamic characterization of the model under investigation several eigenvalues must be extracted. The power method must be extended to look for several eigenvalues with the largest modulus by resorting to an iterative subspace method [13], such as the Arnoldi’s, where, at each step, the algorithm chooses the best initial conditions for equation (7), to find the first $n < N$ eigenvalues with largest modulus. This technique has been employed by Bauchau and Nikishkov to study the stability of periodic motions for large models [14]. Unfortunately, the zero eigenvalues associated with the constraint in the space of the continuous dynamical system, are morphed in eigenvalues on the unit circle in the space of the discrete dynamical system, by means of the inverse of the relation expressed in Eq. (9). Consequently, the first m eigenvalues of a stable problem found by Arnoldi’s method are all related to the constraint, so they do not give any useful insight in the dynamics of the investigated system [15]. Thus, to simplify the analyses, it is necessary to find a method capable of estimating and selecting the dominant eigenvalues of $\Phi(h, k)$, which characterize the phenomenon under investigation, while reducing the computational burden as much as possible. A way to select a reduced set of signals, synthesizing the information related to the dominant transient, must be devised to be able to apply any method for the evaluation of stability properties. These signals can be selected by resorting to a technique capable of extracting spatial coherence in an oscillating system, when the time history of its state variables is known from either a numerical simulations or from the output of several sensors measured in real-life experiments. This method is represented by the Proper Orthogonal Decomposition (POD) used for the analysis of multidimensional data. It provides a way to find the best approximating subspace to a given set of data in a least square sense. The POD allows one to obtain a modal decomposition that is only data dependent and does not assume any prior knowledge of the system. Consequently, it perfectly fits the analysis needs of multidisciplinary models. A reduced order model of a dynamical system can be obtained by simply using a Galerkin projection procedure where the POD modes are used as basis functions. The same ideas can be applied here for the generation of a small subset of significant signals to be used for the identification of the leading eigenvalues characterizing system behavior and stability. For the analysis of mechanical systems, it must be further noticed that these base POMs have an interesting interpretation, since they can be viewed as the result of a least square error optimization for a linear representation of the nonlinear normal modes [7]. By means of the POD all the eigenvalues associated with the constraint will be excluded since there is no work, and consequently no energy, associated with them.

Synthesis by means of POD

Consider a system where all the N state variables are measured at n time steps. Their time averages are usually subtracted from the signals and data are arranged in a $N \times n$ matrix

$$X = [x^{(1)}, x^{(2)}, \dots, x^{(n)}]. \quad (10)$$

An approximation of the system dynamics is obtained by projecting the original N -dimensional state space onto an m -dimensional subspace \mathcal{S} . The main purpose of POD is to find a projection operator Q mapping \mathbb{R}^N onto \mathcal{S} , which minimizes the Euclidean distance of the sampled points from the m -dimensional hyperplane

$$H(Q) = \sum_{i=1}^n \|x^{(i)} - Qx^{(i)}\|. \quad (11)$$

It can be shown (see [5]) that, given $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$, the eigenvalues of the data correlation matrix $E = XX^T$, the optimal m -dimensional projection operator is represented by the $m \times N$ matrix whose rows are the first m eigenvectors of E . To obtain these eigenvectors, assume to compute the the singular value decomposition (SVD) of matrix X^T , i.e.,

$$X = U\Sigma V^T, \quad (12)$$

where Σ is the diagonal matrix of the singular values σ_i , and U , V are unary rectangular matrices. By sorting the singular values σ_i of X in descending order, it can be shown that the matrix $X_m^T = U_m \Sigma_m V_m^T$, where U_m and V_m are the rectangular matrices obtained by retaining the first m columns, and Σ_m is the $m \times m$ principal minor of Σ , is the closest rank m matrix to X in the Frobenius norm. The correlation matrix E can be expressed as:

$$E = XX^T = U\Sigma V^T V \Sigma^T U^T = U\Sigma \Sigma^T U^T. \quad (13)$$

Eq. (13) implies that U is the eigenvector matrix of E , and the corresponding eigenvalues are the squares of the singular values. To choose the dimension m of the approximate subspace that will contain all the significant information we must look at the singular values, since their value expresses the ‘‘signal energy’’ related to the associated POM. These values reach a constant plateau, usually called ‘‘noise floor’’, that characterize the modes which do not contain any significant information [4]. In fact, the use of some form of SVD as a tool to compute the order of a model is a common practice in system identification as well.

If $N \gg n$, then it is more efficient to use the so-called *method of snapshots* in POD literature. It consists in first computing the

matrix V as the eigenvectors matrix of $X^T X$. Once V is known, since

$$XV = U\Sigma,$$

it is clear that the norm of the first m rows of $U\Sigma$ are the singular values, and that these rows, after normalization, are the POMs. Once the POM forms are known, one can obtain their time histories which may be subsequently identified by means of the the AR algorithm of Eq. (8). To obtain a set of signals of comparable amplitude, which will greatly improve the identification phase, it is important to use a set of ‘‘balanced’’ POMs, such as $U\sqrt{\Sigma}$. To obtain a better identification of the resulting signal, it may be useful to employ a second order AR identification algorithm

$$X^{(k+1)} = \Phi_1 X^{(k)} + \Phi_2 X^{(k-1)}, \quad (14)$$

since it is known that mechanical systems are characterized by second order dynamics.

Selection of perturbations

When dealing with systems with a large number of degrees of freedom the choice of an effective set of perturbations may not be a trivial task. If external force impulses are used, it is not possible to safely assert in general that any set of initial impulses will excite all the modes of the system. Often a prior knowledge of the physical phenomena under investigation will help in choosing the perturbation type and level. An advantage of the presented method to find the eigenspaces of the system under investigation, is related to the possibility of choosing the frequency range to be swept. Anyway, a great care must be used in shaping the numeric impulse to obtain the desired results. In fact, usually a numeric impulse will transfer all its energy mainly to the higher frequency segment of the excited modes.

In our extensive experiences, some of which are reported in the followings, no problems were encountered for the selection of correct perturbations. The quality of each identified mode can be assessed by using a ‘‘figure of merit’’ expressed as

$$F_i = \sum_j |\varphi_{ij}| \sigma_j, \quad (15)$$

where φ_{ij} is the j -th component of the i -th eigenvector of the POD state matrix. If F_i is low, the mode is not composed by the dominating POMs, so probably the selected signal may not be the right one for this specific vibration mode.

Anyway, a more general excitation method can be obtained by using white noises inputs $w(t)$, such that $E[w(t)] = 0$ and $E[w(t)w(\tau)^T] = v\delta(t - \tau)$. Assumed that the system is stable

and time invariant, and that all processes are Gaussian, which is true for the cases under investigation, since we are mostly interested in the linearization around an equilibrium orbit, it can be shown that the resulting correlation matrix E is the solution of the following Lyapunov equation [16]

$$AE + EA^T + BvB^T = 0, \quad (16)$$

where A is the usual state-space system matrix, and B is the input matrix which express the connection between the inputs and the internal states. The same equation is used to compute the controllability grammian [17], which contains all the dynamics reachable from the selected inputs, so E is also the controllability grammian. Using a band-limited white noise, which has a flat spectrum over a limited frequency range, it is possible to obtain an approximation of the dominant eigenvalues of the controllability grammian in the desired frequency range. The resulting reduced order model, which will be used in the subsequent identification phase, is thus correlated to what can be obtained by the Balanced Truncation, a classical system reduction technique [17], without resorting to the explicit knowledge of the state-space system matrices A and B . In this case Eq. (8) must be slightly modified to account for the (known) input signals:

$$X^{(k+1)} = \Phi(h, k) X^{(k)} + a^{(k)}, \quad (17)$$

where $a^{(k)}$ represents the stochastic part which has no dependence on the past states and is independent also of $X^{(k)}$, $X^{(k-1)}$, \dots , $X^{(k-n)}$. This technique will be also useful in all the situations where a persistent perturbation may help, such as with highly damped systems.

DYNAMIC CHARACTERIZATION OF ERICA TILTROTOR

The ERICA (Enhanced Rotorcraft Innovative Concept Achievement) project is a tiltrotor aircraft that exploits innovative solutions to overcome some aspects of the tiltrotor design that limit its efficiency in hover and cruise operating conditions. The innovative project is led by Agusta, a Company of AgustaWestland, who proposed five basic research and technological development projects, currently supported by the EU, to assess different systems of the ERICA concept; one of them is the ADYN (Advanced European Tiltrotor Dynamics and Noise) project. The project's partners are the companies Agusta, Eurocopter, Europter Deutschland, IDS; the research institutions CIRA, DLR, NLR, ONERA; the universities NTUA (Greece) and Politecnico di Milano. The analyses are performed by means of heterogeneous simulation codes; as a consequence, the project will also provide cross-code validation.

The analysis technique described in this paper has been applied to the multibody models of the ERICA tiltrotor that are under development at Politecnico di Milano for the ADYN project.

As shown in Figure 1, the aircraft is made of a conventional fuselage, with conventional tail control surfaces. The tilting rotors are carried at the tips of the wing. The rotor nacelles are interconnected by a torque tube that runs inside the wing. The wing itself is divided in two portions, at about half the semi-span. The inner portion is fixed, while the outer one may tilt about its pitch axis. The tilting of the outer portion of wing is independent from that of the rotor nacelles. As a consequence, the movable portion of the wing can assume an angle of attack that is a trade-off between minimizing the rotor downwash effect and maximizing the lift and conventional aircraft controllability during the conversion from hover to forward flight configuration and vice-versa.

The four-bladed rotor is hingeless, with gimballed cyclic flapping motion. The blades have a pronounced tip sweep, and, as usual in tiltrotors, are highly twisted. The arrangement of the control system to allow high flap and twist clearance for four blades requires a careful design; there is a large dependence of the kinematic and structural couplings between flap, lag and twist rotations of the blade on the setting of the collective control.

As a consequence, the multibody approach is particularly suitable for this kind of analysis, since, in presence of appropriately detailed system characterization, it allows to have exact kinematics together with detailed nonlinear flexible component modeling.

Fixed Wing Dynamics

The wing of the aircraft has been modeled with the previously mentioned multibody software and with the Finite Element Analysis code NASTRAN for comparison purposes. The dynamics and the aeroelasticity of the wing are addressed first. The multibody model of the wing is made of beam elements with lumped inertia. A very similar linear finite element model is considered. The multibody model figures are detailed in Table 1. The structure shows different load paths in two parts: the movable portion of the wing, here modeled by a single rigid body, is connected to the torque tube by two flexible actuators and to the fixed portion of the wing by another constraint; the torque tube is connected to the wing by two ribs that constrain the relative transverse displacement, and by a flexible actuator. The fixed portion of the wing is grounded to the fuselage, and symmetric boundary conditions have been applied in the plane of symmetry of the aircraft both for the fixed portion of the wing and for the torque tube. As a consequence, the structure shows some low frequency mode shapes that are typical of classical fixed wing aircraft configurations, i.e. a first beamwise bending mode, a second chordwise bending mode and a third torsional mode which mostly involves a quasi-linear torsion of the tube and a rigid ro-

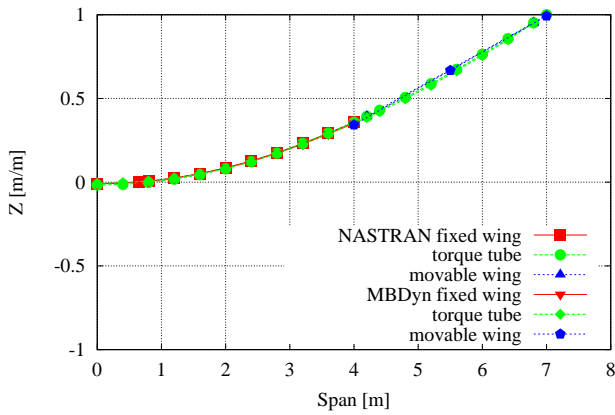


Figure 2. WING MODE 1: BENDING DISPLACEMENTS.

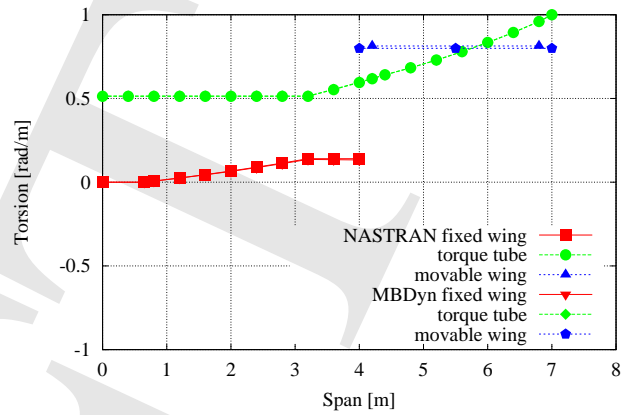


Figure 4. WING MODE 3: TORSION ANGLES.

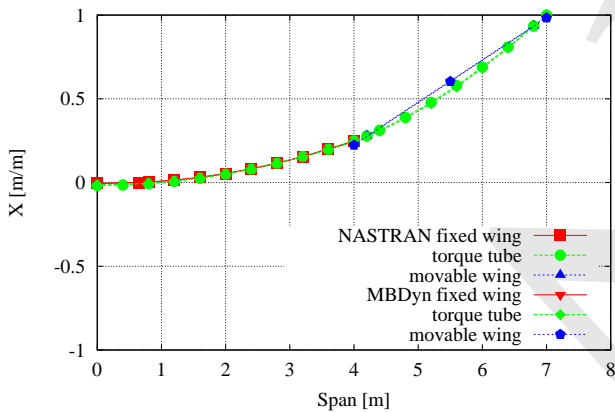


Figure 3. WING MODE 2: IN-PLANE DISPLACEMENTS.

Figures 2– 4.

Rotating Wing Dynamics

The rotor is made of the components listed in Table 1 and it is attached at the tip of the nacelle. **Swasplate.** The shaft rotates about its axis with respect to the nacelle; the shaft and the nacelle respectively carry the fixed and the rotating portion of the swashplate. Each portion is allowed to move along the axis of the shaft and to rotate about two axes orthogonal to the shaft. The fixed plate is connected to the airframe by three flexible actuators that control its attitude to impose the desired collective and cyclic pitch. **Hub and gimbal.** The hub is connected to the shaft by means of an ideal gimbal joint, i.e. a constant velocity joint that allows the tilting of the hub about two mutually orthogonal axes that are orthogonal to the shaft, while transmitting the torque about the shaft axis. This joint is more sophisticated than a universal joint (also known as *Cardano's joint*) and is basically made of two universal joints in series, that are forced to tilt by the same amount.

Trim Point and Excitation. Owing to its peculiar geometrical, structural, and inertial symmetry properties, helicopter rotors, and significantly four-bladed ones, show interesting characteristics. Rotor modes can be divided in three groups: collective (same behavior of all blades), reactionless (same behavior of opposite blades) and cyclic (opposite behavior of opposite blades). Collective and reactionless modes usually are very close, except for those that strongly interact with the flexibility of the control system; in the present case the frequency of the cyclic modes can be much lower than that of the collective ones because of the gimbal degrees of freedom, which allow essentially 1/rev rotor tilting, while the collective and reactionless flapping frequencies are raised by the stiffness of the flexbeam. In any case, the knowledge of the topology of the system allows to separately

tation of the movable portion of the wing. Higher modes involve both second bending modes of the wing and peculiar bending modes of the torque tube.

Trim Point and Excitation. The isolated wing has been analyzed *in vacuo* and with different airstream values up to and well beyond the expected whirl flutter boundary of the tiltrotor aircraft. Most of the analyses have been performed using impulsive excitation, which yields reliable results when the system is slightly damped. The excitation mode required some knowledge of the problem. In fact, a simple exciting force applied at the wing tip, oriented 45° apart in the plane of symmetry of the aircraft and offset from the elastic axis to introduce some torsional moment, was not enough to detect all the frequencies of interest, since the modes involving only the torque tube show very little tip displacement and no rotation. Instead of computing different time series with different excitation points, all the required excitations have been applied in the same analysis; the results match quite well those obtained by means of NASTRAN, as shown in

Table 1. MULTIBODY MODEL PROPERTIES

Component	S	I	B	J	A
Wing					
Fixed wing	13	11	6	5	6
Movable wing	1	1	0	5	1
Torque tube	21	20	10	6	0
Nacelle	0	1	0	0	0
Total	35	33	16	16	7
Total DOFs ¹	402	0	0	37	0
Rotor					
Hub	3	1	0	2	0
Gimbal	0	0	0	5	0
Swashplate ²	4	0	0	7	0
Flexbeam×4	2	2	3	0	0
Cuff×4	2	2	2	4	0
Blade ³ ×4	9	9	4	1	4
Total	59	53	36	34	16
Total DOFs ⁴	654	0	0	89	0

S: structural nodes; I: inertia elements; B: beam elements; J: joints; A: aerodynamic elements

¹The total is 439 DOFs.

²The swashplate also requires a generic element, 6 scalar degrees of freedom and 3 scalar forces.

³Up to 21 structural nodes per blade have been used without significant differences in the frequency range of interest.

⁴Plus 6 scalar DOFs the total is 749 DOFs.

identify these types of modes by using adequate excitation patterns. In the current case, separate runs with collective, reactionless and cyclic impulsive excitation, represented by forces and moments applied at blade tips, are used. All the POD analyses are conducted using the states expressed in the shaft rotating reference frame, in order to obtain the “rotating” rotor modes. First the isolated rotor is analyzed in vacuo to obtain rotating modes and fan plots; then the rotor is analyzed in air. The results of the first set of analyses are shown in Table 2. In the latter case, interesting results have been obtained with realistic persistent band-limited white noise excitation through the control system, which is representative of a possible experimental device for both wind tunnel models and full scale helicopters.

Coupled wing-rotor analyses have been performed as well.

Table 2. ROTOR MODE FREQUENCY IN VACUO

Collective	Cyclic	Reactionless	type
/rev	/rev	/rev	type
1.20	1.01	1.20	flap/gimbal
1.49	1.48	1.49	lead-lag
4.60	3.36	4.60	2 nd bending
7.56	5.42	7.99	torsion
10.91	8.92	11.16	beam/torsion

The analysis in air allowed to highlight a possible instability of the rotor, that is related to the lead-lag mode.

AEROELASTIC STABILITY

The goal of the presented method in this case is the study of the stability of a periodic motion for a nonlinear system of equations. The whole system can be viewed as a non-autonomous nonlinear periodically forced system

$$\dot{x} = f(x, t), \quad f(x, t + T) = f(x, t). \quad (18)$$

By increasing the dimension by one it is possible to transform Eq.(18), into an autonomous system including time as a state variable:

$$\begin{aligned} \dot{x} &= f(x, \theta), \\ \dot{\theta} &= \omega. \end{aligned} \quad (19)$$

It is in this new augmented phase space that a periodic orbit Γ exists, and its stability must be assessed by studying the behavior of Γ -neighboring trajectories. This task may be accomplished going through the determination of the eigenvalues of the monodromy matrix Φ_T , which is the transition matrix that relates two periodic solution state vectors separated by a time period T [18]. This is in general a formidable task, tackled by many authors in the rotorcraft community [19–21], because it requires, for an N -dimension system, the solution of an $(N + N^2)$ -dimension initial value problem, simply to generate the Φ_T matrix. Then it is necessary to compute the eigenvalues of a matrix that in general is full. The presented method, instead, allows to find the dominant eigenspaces of the monodromy matrix from the results of a tailored time simulation. In this case the snapshot matrix is formed joining state vectors sampled with time step T . The additional steps are the same of what has been already presented [15]. As a result, it is possible to follow only the path

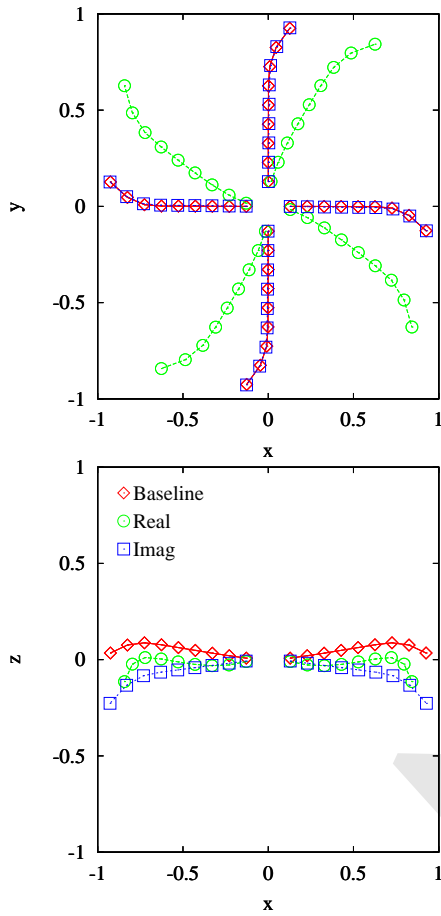


Figure 5. ISOLATED ROTOR UNSTABLE MODE. ABOVE: TOP VIEW; BELOW: BLADE 1-3 FRONT VIEW.

of the dominant eigenvalues, which characterize the system response, substantially reducing the computational burden.

Tiltrotor Periodic Stability

The isolated rotor potential instability mainly appears in form of a collective lead-lag mode, with some participation of blade flap. This does not mean that the unstable mode is collective only, because the dominating collective response could be determined by the significant collective excitation resulting from the rotor wind-up. The unstable mode form is shown in Figure 5.

Nevertheless, the coupled rotor-wing system shows an analogous lead-lag instability which is basically a reactionless mode, as shown in Figure 6. While collective and cyclic modes can interact with the dynamics of the wing, the reactionless ones cannot. One possible explanation is that the wing dynamics damps out the collective and cyclic lead-lag movement, so the remaining reactionless portion of the lead-lag mode dominates the response. Figure 7 shows some of the computed eigenvalues for different configurations. The system is clearly passing through

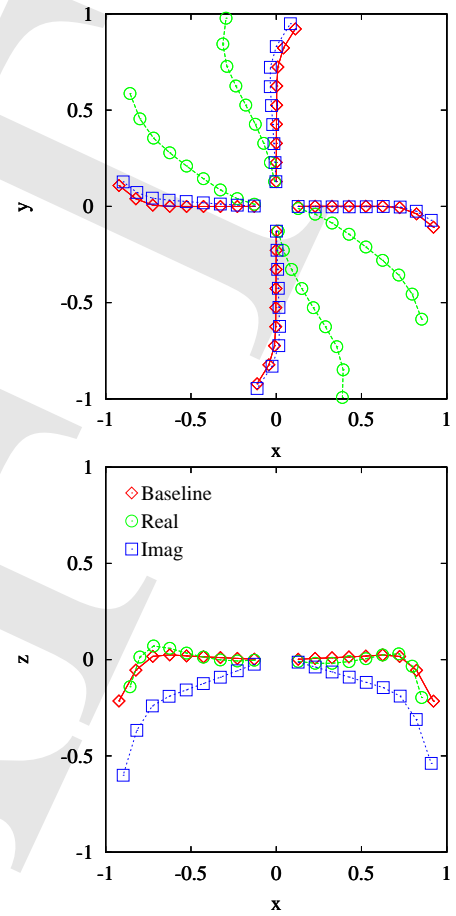


Figure 6. WING-ROTOR UNSTABLE MODE. ABOVE: TOP VIEW; BELOW: BLADE 1-3 FRONT VIEW.

an Hopf bifurcation, which cause the rise of the instability of the periodic movement.

CONCLUSIONS

The complexity of the dynamic characterization phase for large multibody nonlinear models has been highlighted. A method has been presented to reduce the computational burden and to obtain few global parameters which well represent the overall system behavior. As shown with the tiltrotor example, the user has an high capability of tailoring the analysis to narrow the research around the interesting ranges. Furthermore, the same methodology has been applied, with no additional cost, to the analysis of the stability of periodic motions.

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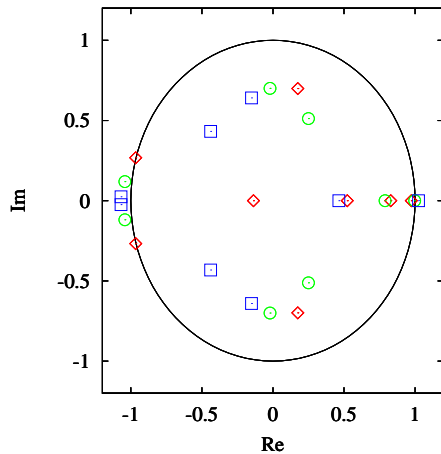


Figure 7. WING-ROTOR PERIODIC EIGENVALUES.

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REFERENCES

- [1] Quaranta, G., Masarati, P., and Mantegazza, P., 2002. "Multibody analysis of controlled aeroelastic systems on parallel computers". *Multibody Systems Dynamics*, **8** (1), pp. 71–102.
- [2] Schiehlen, W., 1990. *Multibody Systems Handbook*. Springer-Verlag, Berlin.
- [3] Brenan, K. E., Campbell, S. L. V., and Petzold, L. R., 1989. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*. North-Holland, New York.
- [4] Ravindra, B., 1999. "Comments on "On the physical interpretation of proper orthogonal modes in vibrations"". *Journal of Sound and Vibration*, **219** (1), pp. 189–192.
- [5] Holmes, P., Lumley, J., and Berkooz, G., 1996. *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*. Cambridge University Press.
- [6] Epureanu, B., Dowell, E., and Hall, K., 2001. "Reduced-order models of unsteady viscous flows in turbomachinery using viscous-inviscid coupling". *Journal of Fluids and Structures*, **15**, pp. 255–273.
- [7] Feeny, B., and Kappagantu, R., 1998. "On the physical interpretation of proper orthogonal modes in vibrations". *Journal of Sound and Vibration*, **211** (4), pp. 607–616.
- [8] Azeez, M., and Vakakis, A., 2001. "Proper orthogonal decomposition (POD) of a class of vibroimpact oscillations". *Journal of Sound and Vibration*, **240** (5), pp. 859–889.
- [9] Ruotolo, R., and Surace, C., 1999. "Using SVD to detect damage detection in structures with different operational conditions". *Journal of Sound and Vibration*, **226** (3), pp. 425–439.
- [10] Nannoni, F., Giancamilli, G., and Cicalè, M., 2001. "ERICA: the european advanced tiltrotor". In *Proceeding of the 27th European Rotorcraft Forum*, pp. 55.1–55.15.
- [11] Hairer, E., and Wanner, G., 1996. *Solving Ordinary Differential Equations II. Stiff and Differential Algebraic Problems*. Springer-Verlag, Berlin, Germany. 2nd rev. ed.
- [12] Masarati, P., Lanz, M., and Mantegazza, P., 2001. "Multi-step integration of ordinary, stiff and differential-algebraic problems for multibody dynamics applications". In *XVI Congresso Nazionale AIDAA*.
- [13] Saad, Y., 1992. *Numerical Methods for Large Eigenvalue Problems*. Manchester University Press, Manchester, England, UK.
- [14] Bauchau, O. A., and Nikishkov, Y. G., 2001. "An implicit transition matrix approach to stability analysis of flexible multi-body systems". *Multibody Systems Dynamics*, **5** (3), pp. 279–301.
- [15] Quaranta, G., Mantegazza, P., and Masarati, P., Assessing the local stability of periodic motions for large multibody nonlinear systems using POD. Accepted for publication on *Journal of Sound and Vibration*.
- [16] Friedland, B., 1987. *Control System Design. An Introduction to State Space Methods*. McGraw Hill, New York, NY.
- [17] Van Dooren, P., 2000. "Grammian based model reduction of large-scale dynamical systems". *Numerical Analysis 1999*, pp. 231–247.
- [18] Seydel, R., 1994. *Practical Bifurcation and Stability Analysis. From Equilibrium to Chaos*. Springer-Verlag, New York.
- [19] Friedmann, P., Hammond, C., and Woo, T.-H., 1977. "Efficient numerical treatment of periodic systems with application to stability problems". *International Journal for Numerical Methods in Engineering*, **11**, pp. 1117–1136.
- [20] Peters, D. A., 1994. "Fast floquet theory and trim for multibladed rotorcraft". *Journal of the American Helicopter Society*, **39** (4), pp. 82–89.
- [21] Subramanian, S., and Gaonkar, G. H., 1996. "Parallel fast-floquet analysis of trim and stability for large helicopter models". In *22nd European Rotorcraft Forum*, pp. 94.1–94.14.