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A New Class of Staggered Semi-Implicit Discontinuous Galerkin Methods for Fluid and Geophysical Flows

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Abstract

In this seminar we present a new class of well-balanced, arbitrary high order accurate in space and time semi-implicit discontinuous Galerkin methods for the solution of fluid and geophysical flows on unstructured staggered meshes. This class of methods was applied to several PDE systems such as shallow water, incompressible Navier-Stokes, compressible Euler / Navier-Stokes, natural convection problems and linear elasto-dynamics in two and three-space dimensions. Isoparametric finite elements are used to take into account curved domain boundaries.

Regarding incompressible Navier-Stokes equations, the discrete pressure is defined on the main (tri-angular or tetrahedral) grid and the velocity field is defined on the edge-based staggered grid.

High order in time can be achieved by using a space-time finite element framework, where the basis and test functions are piecewise polynomials in both space and time. Formal substitution of the discrete momentum equation on the dual grid into the discrete continuity equation on the primary grid yields a very sparse system for the scalar pressure involving only the direct neighbor elements, so that it becomes a block four-point system in 2D and a block five-point system for 3D tetrahedral meshes. The resulting linear system is conveniently solved with a matrix-free GMRES algorithm. A very simple and efficient Picard iteration is then used in order to derive a space-time pressure correction algorithm that achieves also high order of accuracy in time. The special case of high order in space low order in time allows us to recover further regularity about the main linear system for the pressure, such as the symmetry and the positive semi-definiteness in the general case. This allows us to use a very fast linear solver such as the conjugate gradient (CG) method. The flexibility and accuracy of high order space-time DG methods on curved unstructured meshes allows to discretize even complex physical domains with very coarse grids in both space and time. Similar reasoning can be applied to other PDE systems such as compressible Euler/Navier-Stokes, natural convection problems, shallow water and linear elasticity.

This class of numerical methods is limited by a CFL condition due to an explicit treatment of the nonlinear convective term. However, this condition is based on the fluid velocity and not on the sound speed. This make the method particularly interesting for low Mach number flows. A combination of artificial viscosity and a Multi-dimensional Optimal Order Detection (MOOD) technique allows to simulate also shock wavers and high Mach number flows.

This new family of numerical schemes were verified by solving a series of typical numerical test problems and by comparing the obtained numerical results with available exact analytical solutions or other numerical reference data.

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